

## Retail Banking Loan Portfolio Equilibrium Mix : A Markov Chain Model Analysis

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**Abstract:** The variance analysis of actual loan sanctions with the non-documented method of loan allocation of the selected retail bank, over a period of 24 months, revealed that there is a scope to improve their income earnings. Realizing its importance Markov Chain Market Share model was applied to inter temporal data of loan disbursements of the selected bank. By applying Estimate Transition Matrix, scope for probability of loan switching among its types was calculated to suggest the probable mix of loan portfolio. From the results it was suggested that the loan proportions among various types were as follows: Housing (32.0 %), Others (28.1 %), Business (20.0 %) and Education (19.7 %). These proportions can be taken as guideline percentage within the government norms for the priority sector. Simulation studies were also done to calculate the expected income of interest using Markov proportions and compared with the actual interest earnings to prove the superiority of the model.

**Keywords:** Markov Chain Market Share Model, Loan Portfolio, Model Superiority

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### INTRODUCTION

Over the past one decade the banking industry in Malaysia had gone through many structural changes in terms of increase in branch network, provision of wide range of banking services and acceleration of credit activities in different ways [1]. The financial crisis in 1997-98 has created a tremendous pressure in the banking sector which was sorted out by means of consolidation process carried out by the Bank Negara Malaysia, the Malaysian Central Bank. The Central Bank envisaged the merger schemes to combat the crisis and termed some of the merged banks as anchor banks, to accelerate the economic growth. The survival of any banking sector normally depends on their ability to improve their efficiency and effectiveness in their product offerings [2]. [3] explains that there are three main banking objectives.: (1) It has to ensure that its business should run as usual by ensuring that its debts do not exceed its liability. (2) The bank must maintain its liquidity; i.e the bank should be able to meet withdrawals at any given point of time. Finally, the bank has to generate profits for the stockholders (profitability)[3]. Thus, the bank should maintain an appropriate funds portfolio for their survival and growth. The variance of the actual loan sanctions and its allocation over a period of 24 months in the retail bank selected for the study revealed two important findings [4]. Firstly, the loan allocation policy adopted by the bank management is suspectingly based on non documented hybrid model. Secondly, the switching of loan allocation from one type to another is also

possible. These two findings suggested that there should be a systematic method of loan portfolio management, in order to maximize the interest income of the bank [5]. The current study attempts to devise a loan allocation policy to different type of loans using Markov Chain Market Share Model. Kosubud and Stokes suggested that Markov Chain application in the business situation application is rich in terms of economics and policy implications [6]. In this study an attempt has been made to estimate the transition matrix using inter-temporal data on loan disbursements. This provides the probability of loan switching among its types [7]. Simulation process was also carried out to calculate the expected income of interest from all loan types using the actual loan disbursement data and Markov proportions to evaluate the superiority of Markov Chain approach.

**Bank Loan Portfolio:** The bank loan portfolio of a selected bank is composed of three main strategic business operations namely Retail Banking, Business Banking and Corporate Banking with an individual share of 39, 28 and 33% respectively as at January 2002[8 , 9]. The Business Banking caters to small and medium-sized companies (paid up capital up to RM1.0 million) and generally concentrates on business loan and trade financing related to their business. The Corporate Banking, serves to the top-tier Malaysian conglomerates or corporate sector of listed or about to be listed in the Kuala Lumpur Stock Exchange (KLSE), in the form of loans such as revolving credits, huge capital expenditure loan, bridging loan, multi-million

project undertakings by way of either term loan, overdraft or floating rate loan and other package of trade finance. The retail banking emphasizes on individual customer loans like housing, small business (to sole-proprietors, partnership or small size companies with paid-up capital up to RM 250,000), education and miscellaneous loans such as staff housing, trust receipt, purchase of Amanah Saham Bumiputra (ASB) certificates and personal overdraft facility [10]. The later miscellaneous loans are classified as "other loan". In the current study the focus was made only on retail banking unit. The reason for selecting the retail banking was on two folds. (1) The retail loan portfolio is usually greater than the other portfolios. (2) Retail loan products are generally popular in branch banking networks.

### MATERIALS AND METHODS

The bank chosen for the current study was the second largest in the local banking sector in Malaysia. The identity of the bank is not disclosed for secrecy reasons under Banking and Financial Institutions (BAFIA) Act, 1989 of Malaysia [11]. Its asset position crosses more than RM. 70.0 billion with over 250 branches and 798 ATM network throughout the country. The bank recorded a profit before tax (PBT) amounting to RM 648.7 million at the group level and RM.250.8 million at the bank level as on Dec.2000 [12]. The period of the study was restricted to 24 months beginning from January 2000, since the information prior to that period would not represent the loan disbursement trend of the bank because it was one of the merged bank under the merger scheme of the Bank Negara Malaysia[13]. The study aimed to devise the determination of equilibrium loan allocation using Markov Chain Market Share Model [14 , 15]. The main objective of this study was aimed towards obtaining optimal loan allocation mix policy, which could be used as guiding principle on future allocation purpose. Time series data on four loan disbursements were used as a basis of estimating a transition probability matrix.(TPM). Transition probability matrix gives the probability of loan switching from one type to another type. Markov chain model was then applied to the set of time series on the actual loan disbursement proportions to calculate the estimated TPM using a quadratic programming technique [16]. Further, several statistical tests were conducted to investigate the suitability of using macro data, homogeneity, communicability, periodicity and absorption status of the process. Forecast of loan disbursement for a period of 12 months was then made, in order to forecast the future allocation of each type of loan.

**Markov Probability Model:** The probability of switching a loan disbursement from loan type  $i$  to loan

type  $j$  is a conditional probability and can be represented by the transition matrix  $P = [p_{ij}]$  such that

$$\sum_{j=1}^m p_{ij} = 1.$$

Indices  $i$  refers to the number of loan

type. For example  $p_{2,1}$  represents the probability of a change in loan disbursement from business to housing in the next period of time. While  $p_{i,i}$  represents the probability of no change in loan disbursement for loan type  $i$ . The stochastic model used to explain the loan disbursement behavior is a Markov Chain with finite number of States  $\{E\}$  Markov process  $\{X_t\}$  with discrete time  $t$  such that  $p_{ij}$  in general represents the probability of the process moving from state  $i$  at time  $t-1$  to state  $j$  at time  $t$ . In this study we assume that the loan disbursement for type  $i$  in the next period  $t$  (month) is only determined by the loan disbursement at the preceding period  $t-1$ . In other words, the "history" of loan disbursement before time  $t-1$  does not influence the future loan disbursement. This is known as a first order time dependency. In statistical notation it is represented by

$$P(X_t = j | X_0, X_1 \dots X_{t-1} = i) = P(X_t = j | X_{t-1} = i) \text{ for } i, j \in E$$

Furthermore, it is also assumed that the underlying variable that are responsible for the generation of loan disbursement do not change overtime, such that the transition probability has a stationary property i.e.

$$P(X_t = i | X_{t-1} = i) = p_{ij}(t + 1) = p_{ij} \text{ for all } t.$$

Furthermore, the probability relations must also be satisfied

$$\sum_{j=1}^m p_{ij} = 1 \text{ and } 0 \leq p_{ij} \leq 1 \text{ for all } i \text{ and } j \in E$$

**Estimation of Probability Transition Matrix:** The estimation of the probability transition matrix plays a major and crucial role in the study of a Markov process [17]. If a process that follows a known probability distribution, the estimation can be made with less difficulty, otherwise the estimation procedure is a problem oriented. For micro economic data that traces the movement from any given state to another states, then, the estimation procedure follows that of a multinomial distribution, that is  $p_{ij} = \frac{n_{ij}}{n_i}$ , where  $n_{ij}$  is

the number of time the process moves from state  $i$  to state  $j$  and  $n_i$  is the number of time the process is in state  $i$ . However for the macro economic data the estimation procedure is quite tedious. A comprehensive survey of estimation techniques of the stationary transition matrix using macro data is provided by [18]. Among the several techniques considered, Bayesian estimation is the best, but among the non Bayesian, they proposed the following ranking: maximum likelihood (MLE), weighted least squares, unweighted

restricted least square, minimum absolute deviation and the unrestricted least square estimator. In this study, however, the estimation of the transition matrix is made by using the unweighted restricted using the ordinary least square techniques.

Following Lee, Judge and Zellner, the first order conditional probability can be rewritten as

$$P(X_t = j) = \sum_{i=1}^m P(X_t = j | X_{t-1} = i) = \sum_{i=1}^m P(X_{t-1} = i | X_t = j) P(X_{t-1} = i)$$

or

$$q_j(t) = \sum_{i=1}^m q_i(t-1) p_{i,j}, \text{ where } q_j(\cdot) \text{ and } P(\cdot) \text{ represent}$$

the unconditional probability. If  $q_j(t)$  is replaced by the observed proportion  $y_j(t)$ , then the sample observation may be assumed to be generated by the following stochastic relation.

$$y_j(t) = \sum_{i=1}^m y_i(t-1) p_{i,j} + u_j(t)$$

or

$$Y_j = X_j P_j + U_j$$

where  $Y_j$ ,  $X_j$ , and  $P_j$  are defined as follows.  $Y_j$  is a vector of proportion for loan type  $j$ .  $X$  with  $(t-1)$  components  $X_j$  is a matrix of proportion with dimension of  $(t-1)$  by  $m$ .  $P_j$  is a probability vector ( $p_{ij}$  for  $i = 1, 2, 3, \dots, m$ ).  $U_j$  is a vector of random error. Similarly, for all  $i$  and  $j$  the possible movements of the process are described in the following equation.

$$Y = XP + U, \text{ where}$$

$$Y' = [Y'_1, Y'_2, \dots, Y'_m], P' = [P'_1, P'_2, \dots, P'_m], U' = [U'_1, U'_2, \dots, U'_m]$$

and  $X$  is a block diagonal matrix with  $X_1 = X_2 = \dots = X_m$ . Thus the above equation is used to estimate  $P$  by the ordinary least square (OLS) technique subject to the non negativity and equality constraints; i.e.

$$\min [U'U = (Y - XP)'(Y - XP)]$$

such that  $GP = 1 \quad P \geq 0$ ,

Where  $G = [I_1, I_2, \dots, I_m]$  and  $I_j$  is the identify matrix. This optimization problem can be solved by the quadratic programming routine provided that  $(X'X)$  is non-singular. Under this formulation however, the error terms are not uncorrelated, thus  $P$  (the estimated  $P$ ) is an unbiased but consistent estimator of  $P$  [19].

### Estimating the Transition Probability Matrix for the Loan Portfolio:

Monthly time series data on actual loan disbursement for a period of 24 months beginning January 2000 ( $t = 1$ ) for four types of loan are used as a basis to estimate the transition probability matrix. Following the estimation procedure discussed earlier, we need to define the appropriate vectors and matrix before an optimization routine can be applied. Since  $y_j(t)$  is defined as a proportion of loan type  $j$ , at time  $t$ , then the actual loan disbursements have to be changed into proportion. This can be done by dividing the

individual actual loan disbursement by its total actual loan disbursement for each period  $t$  (Table 1).

**Definition of Vectors and Matrix:** The stochastic relation  $y_j(t) = \sum_{i=1}^m y_i(t-1) p_{ij} + u_j(t)$  is used to estimate

the transition probability matrix. The proportion of loan disbursement type  $j$  at time  $t$  is the summation of the product of all loan proportions at time  $(t-1)$  and its probability  $p_{ij}$  over all types of loan. In terms of matrix, that relation is equivalent to  $Y_j = X_j P_j + U_j$ . Thus for

each loan type  $j$ ,  $Y_j$ ,  $X_j$  and  $P_j$  are defined as follows.

For  $j = 1$ ,  $Y_i$  is a 23 component vector of proportion for loan type 1 beginning from  $t = 2$  to  $t = 24$  (December 2001). Similarly for loan type 2, 3 and 4,  $Y_j$  is defined accordingly. Matrix  $X_j$  is a  $23 \times 4$  matrix of loan proportions beginning at  $t = 1$ , to  $t = 23$ .  $P_j$  is a probability vector of  $p_{ij}$  for all  $i \in E$ . Thus each relation of  $Y_j = X_j P_j + U_j$  will give the estimates of the probability of loan switching from each type  $i$  to type  $j$ . If all possible switchings of several loan types are cast in one aggregate model, (for  $i$  and  $j \in E$ ), then model  $Y = XP + U$  is used with respective vector  $Y$  and matrix  $X$  and  $P$  are defined accordingly.

Using the estimation technique, discussed earlier the estimated  $P$  matrix is obtained by minimizing the summation of sum of square of the error terms, with probability constraints attached. Thus the estimation procedure follows that of quadratic programming model that is

$$\text{Min } U'U = (Y - XP)'(Y - XP) = Y'Y - 2XP'Y + P(XP)'$$

Such that  $GP = 1 \quad P \geq 0$

In this study, two computer packages namely SHAZAM for calculating the inputs to the objective function ( $-2X'Y$  and  $X'X$ ) and the CPLEX routine for solving the quadratic programming problems were [20] used. Thus upon defining the various vectors and matrix according to the format of the CPLEX routine, optimal solution to  $P$  is finally obtained. Values for vector  $Y$  and matrix  $X$ ,  $G$ ,  $P$ ,  $-2X'Y$  and  $2X'X$  are accordingly defined. Decision variables indicated by variables  $x_1, x_2, \dots, x_{16}$  represent the probability variables with the following mapping.

Decision variable	Probability variable	Decision variable	Probability variable
X1	P11	X9	P13
X2	P21	X10	P23
X3	P31	X11	P33
X4	P41	X12	P43
X5	P12	X13	P14
X6	P22	X14	P24
X7	P32	X15	P34
X8	P42	X16	P44

Table 1: Actual Loan Disbursement and Proportion For Four Loan Types (RM'000)

Month/ Loan Type	Housing		Business		Education		Others		Total
	Actual	Proportion	Actual	Proportion	Actual	Proportion	Actual	Proportion	
Jan – 00	6,226,494.79	0.487	826,616.40	0.065	783,045.10	0.061	4,947,335.53	0.387	12,783,491.82
Feb – 00	7,696,332.52	0.655	689,182.50	0.059	725,801.80	0.062	2,646,161.01	0.225	11,757,477.83
Mar – 00	7,278,007.17	0.619	705,378.29	0.060	733,785.62	0.062	3,044,302.94	0.259	11,761,474.02
Apr – 00	7,359,520.85	0.619	721,954.68	0.061	741,857.26	0.062	3,070,457.81	0.258	11,893,790.60
May – 00	7,441,947.49	0.619	738,920.61	0.061	750,017.69	0.062	3,096,709.96	0.257	12,027,595.75
Jun – 00	7,525,297.30	0.619	764,493.70	0.063	758,267.89	0.062	3,114,847.32	0.256	12,162,906.20
Jul – 00	7,609,580.63	0.619	790,955.04	0.064	766,608.83	0.062	3,132,594.39	0.255	12,299,738.89
Aug – 00	7,694,807.93	0.619	818,335.58	0.066	593,501.72	0.048	3,331,465.73	0.268	12,438,110.96
Sept – 00	7,780,989.78	0.619	846,667.32	0.067	599,436.74	0.048	3,350,945.87	0.266	12,578,039.70
Oct – 00	7,868,136.86	0.604	875,983.39	0.067	605,431.10	0.047	3,668,719.74	0.282	13,018,271.09
Nov – 00	7,956,260.00	0.590	906,318.06	0.067	611,485.42	0.045	3,999,847.11	0.297	13,473,910.58
Dec – 00	8,045,370.11	0.577	937,706.81	0.067	617,600.27	0.044	4,344,820.26	0.312	13,945,497.45
Jan – 01	6,782,094.20	0.515	811,790.50	0.062	856,491.50	0.065	4,710,349.61	0.358	13,160,725.81
Feb – 01	8,881,527.15	0.653	872,222.40	0.064	1,415,487.60	0.104	2,441,875.75	0.179	13,611,112.90
Mar – 01	7,646,795.90	0.567	806,081.40	0.060	1,112,173.90	0.082	3,926,174.80	0.291	13,491,226.00
Apr – 01	7,641,660.25	0.558	1,196,347.40	0.087	915,116.78	0.067	3,936,109.57	0.288	13,689,234.00
May – 01	7,737,181.00	0.559	1,211,301.74	0.088	935,706.91	0.068	3,959,048.23	0.286	13,843,237.88
Jun – 01	7,833,895.77	0.560	1,226,443.01	0.088	956,760.31	0.068	3,981,875.22	0.284	13,998,974.31
Jul – 01	7,931,819.46	0.560	1,241,773.55	0.088	978,287.42	0.069	4,004,582.33	0.283	14,156,462.77
Aug – 01	8,030,967.21	0.561	1,257,295.72	0.088	1,000,298.89	0.070	4,027,161.16	0.281	14,315,722.98
Sept – 01	8,131,354.30	0.562	1,273,011.92	0.088	1,022,805.61	0.071	4,049,603.03	0.280	14,476,774.86
Oct – 01	8,232,996.22	0.549	1,288,924.57	0.086	1,045,818.74	0.070	4,415,722.45	0.295	14,983,461.98
Nov – 01	8,335,908.68	0.538	1,305,036.12	0.084	1,069,349.66	0.069	4,797,588.69	0.309	15,507,883.15
Dec – 01	8,440,107.45	0.526	1,321,349.08	0.082	1,093,410.03	0.068	5,195,792.42	0.324	16,050,659.06

The constraint names  $C_1, C_2, C_3$  and  $C_4$  represent the probability constraints where

$$C_1 = \sum_{j=1}^4 p_{1j} = 1, C_2 = \sum_{j=1}^4 p_{2j} = 1, C_3 = \sum_{j=1}^4 p_{3j} = 1$$

and  $\sum_{j=1}^4 p_{4,j} = 1$ .

The optimal objective value  $z = -2.0738$ . Since this programming model consists of only 16 decision variables and 4 constraints, the solution time is quite negligible using Mathematical Programming System CPLEX solver.

**RESULTS AND DISCUSSION**

**Transition Probability Matrix:** The transition probability matrix for the loan portfolio is given in Table 2 while Fig. 1 represents its pictorial representation. The transition probability matrix shows that the probability of loan switching from business to other loan is quite high (0.736) while loan switching from housing to education is low (0.116). Probability of no loan switching is quite high for education loan (0.526) while probability of no loan switching is very low for other loan (0.155). Loan switching from business to housing, business to education and education to other cannot be made in one time period due to its zero probability. Loan switching to housing loan is relatively high from other loan (0.478) but relatively low from education loan (0.183). One important observation could be highlighted. With non zero probability loan switching will take place from any other loan to business loan indicating that business loan allocation is not fully utilized. The interpretation of this probability values should be made cautiously. Firstly, the probability value gives us the indication of loan switching. It may actually affect the switching or it may not be. If it affects the switching then the probability value gives the probability of switching to other loan types. Secondly, the probability value

Table 2: Transition Probability Matrix

	<i>HS</i>	<i>BS</i>	<i>ED</i>	<i>OT</i>
<i>HS</i>	0.465	0.131	0.116	0.288
<i>BS</i>	0.0	0.264	0.0	0.736
<i>ED</i>	0.183	0.291	0.526	0.0
<i>OT</i>	0.478	0.169	0.198	0.155

also indicates that if a bank receives a loan application (say a housing loan), then if its allocation is still available, then there is no switching. Otherwise, loan switching is made. The probability value gives the

probability of 0.465 no switching, 0.131 of switching to business loan, 0.116 of switching to education loan and 0.288 of switching to other loan. Other probability values should be interpreted accordingly. The pictorial representation indicates the switching of loan derived from the transition probability matrix. A directed arch represents the non zero probability of switching from one type to another type [21]. As indicated in earlier section, the use of micro economic data that trace the loan switching from various types is preferred.

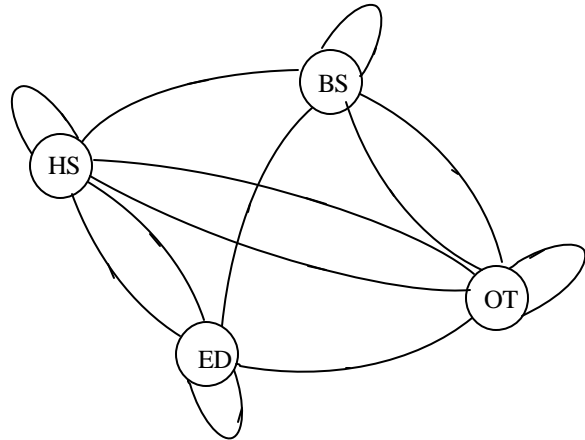


Fig. 1: Pictorial Representation of the Transition Probability Matrix

**Legend**

- HS : Housing Loan
- BS : Business Loan
- ED : Education Loan
- OT : Other Loan

**Stationarity/Homogeneity of The Process:** For useful application of the Markov process in particular to business and economic problems, one has to further investigate the stationarity of the process. By stationarity we mean that the underlying factors that are responsible for the generation of the data do not change significantly over the sampling period (data collection time) and the forecast periods. This could be verified by analyzing the trend of the backcast proportion of the loan disbursements and consequently conducting the Chi-square test of homogeneity [22 and 23]. If the backcast proportions for all loan types do not exhibit an erratic movement, then one would conclude that the proportions are stable. Both types of homogeneity analyses use of the estimated transition probability matrix: backcast values and theoretical transition probability distribution of the process for the Chi-square test. The underlying assumption is that movement of the process is governed by the estimated transition probability matrix; as such upon fulfilling the homogeneity criteria, the transition

Table 3: Actual and Backcast Proportion of Loan Disbursement

Month/ Loan Type	Housing		Business		Education		Other Loan	
	Actual	Backcast	Actual	Backcast	Actual	Backcast	Actual	Backcast
Jan – 00	0.4871	-	0.0647	-	0.0613	-	0.3870	-
Feb – 00	0.6546	0.4232	0.0586	0.1639	0.0617	0.1657	0.2251	0.2473
Mar – 00	0.6188	0.4239	0.0600	0.1570	0.0624	0.1531	0.2588	0.2661
Apr – 00	0.6188	0.4234	0.0607	0.1586	0.0624	0.1560	0.2582	0.2620
May – 00	0.6187	0.4231	0.0614	0.1586	0.0624	0.1559	0.2575	0.2624
Jun – 00	0.6187	0.4228	0.0629	0.1587	0.0623	0.1558	0.2561	0.2628
Jul – 00	0.6187	0.4221	0.0643	0.1588	0.0623	0.1554	0.2547	0.2637
Aug – 00	0.6186	0.4214	0.0658	0.1590	0.0477	0.1551	0.2678	0.2645
Sept – 00	0.6186	0.4250	0.0673	0.1573	0.0477	0.1501	0.2664	0.2676
Oct – 00	0.6044	0.4234	0.0673	0.1575	0.0465	0.1498	0.2818	0.2685
Nov – 00	0.5905	0.4248	0.0673	0.1579	0.0454	0.1506	0.2969	0.2668
Dec – 00	0.5769	0.4254	0.0672	0.1583	0.0443	0.1514	0.3116	0.2651
Jan – 01	0.5153	0.4259	0.0617	0.1586	0.0651	0.1521	0.3579	0.2634
Feb – 01	0.6525	0.4231	0.0641	0.1630	0.1040	0.1652	0.1794	0.2487
Mar – 01	0.5668	0.4087	0.0597	0.1627	0.0824	0.1661	0.2910	0.2625
Apr – 01	0.5582	0.4183	0.0874	0.1629	0.0668	0.1669	0.2875	0.2518
May – 01	0.5589	0.4097	0.0875	0.1640	0.0676	0.1570	0.2860	0.2691
Jun – 01	0.5596	0.4095	0.0876	0.1641	0.0683	0.1572	0.2844	0.2692
Jul – 01	0.5603	0.4092	0.0877	0.1641	0.0691	0.1574	0.2829	0.2692
Aug – 01	0.5610	0.4089	0.0878	0.1642	0.0699	0.1576	0.2813	0.2693
Sept – 01	0.5617	0.4086	0.0879	0.1643	0.0707	0.1578	0.2797	0.2693
Oct – 01	0.5495	0.4083	0.0860	0.1644	0.0698	0.1579	0.2947	0.2693
Nov – 01	0.5375	0.4097	0.0842	0.1646	0.0690	0.1590	0.3094	0.2667
Dec – 01	0.5258	0.4110	0.0823	0.1648	0.0681	0.1601	0.3237	0.2642

probability matrix at least from the statistical point of view actually describes the loan disbursement process

**Trend Analysis of the Backcast Proportion and the Chi-square Test:** Values of the backcast proportion or one period forecast proportion of the individual type of loan disbursement are estimated from the following matrix operation.

$$\hat{X}(t+1) = X(t)P$$

where  $\hat{X}(t+1)$  and  $X(t)$  are vectors of the backcast proportion and the actual proportion for all past values of  $t$  respectively. Table 3 gives the value of actual and backcast proportions of the loan disbursements. Fig. 2 to 5 show the trend of actual and backcast proportion. It is observed from Table 3 that the housing loan proportion has a decreasing trend while the proportions of business, education and other loans has an increasing trend. The same phenomena is also basically observed for the backcast proportion. Though the trend for the actual and backcast proportion seems to be consistent, the actual proportion in particular the other loan has a fluctuating movement. However for the backcast proportion, the trend is quite smooth which connotes a stable trend. Thus one would conclude that the estimated transition matrix produces a stable trajectory which will imply homogeneity. Had the trend for the backcast proportion exhibit an erratic movement, then one would obviously conclude that the underlying factors that are responsible for the generation of the data had changed the loan process significantly.

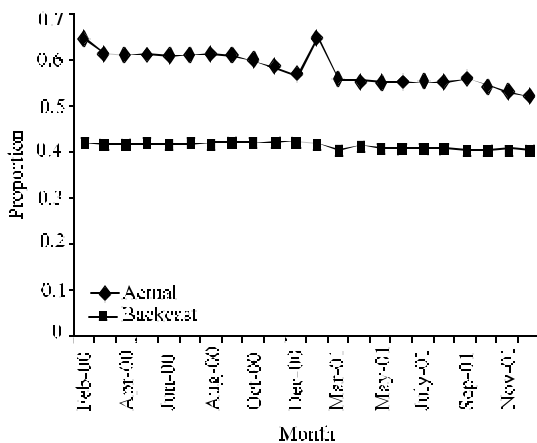


Fig. 2: Housing Loan: Actual and Backcast Proportion

Calculation of the Chi-square Statistics for the proportion is made and with a 5 % significant level, the test concludes that the transition probability matrix obtained from the loan disbursement data describe the population theoretical probability of loan switching.

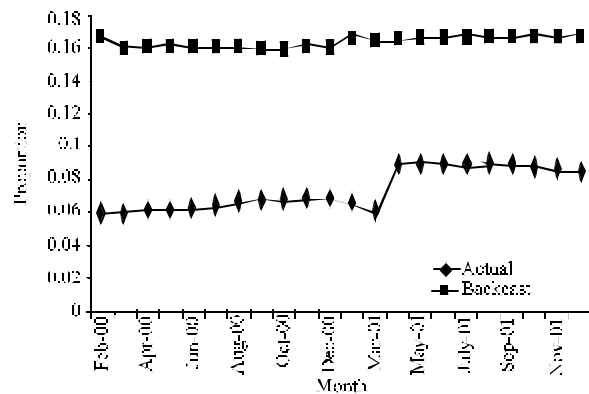


Fig. 3: Business Loan – Actual and Backcast Portfolio

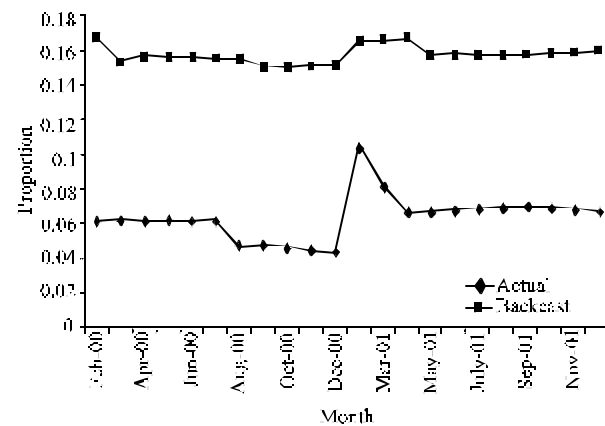


Fig. 4: Education Loan – Actual and Backcast Proportion

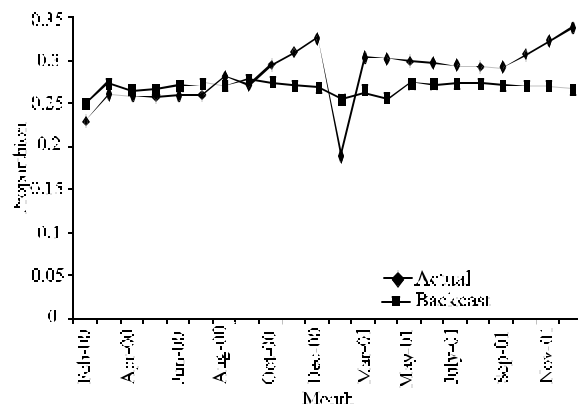


Fig. 5: Other Loan – Actual and Backcast Proportion

**Steady State Distribution and The First Passage Time:** In practice, steady state distribution will indicate the long term proportion of the loan disbursements which in turn be used to estimate the optimal loan portfolio mix.

In this study, the estimated steady state distribution is given as follows:

$$\pi = \begin{matrix} & \text{HS} & \text{BS} & \text{ED} & \text{OT} \\ \pi & = & 0.320 & 0.20 & 0.197 & 0.283 \end{matrix}$$

Thus in the long run, the housing loan should constitute 32% of the total loan, 20% for the business loan, 19.7% for the education loan and 28.3% for the other loan. This information also indirectly indicates the relative importance of the various loan type, besides the information on  $M_{ij}$  the first passage time. By performing the matrix operation on  $Z = [I - \pi + P]^{-1}$  and applying the formula for  $M_{ij}$ , the matrix of first passage time  $M$  is estimated as follows:

$$M = \begin{matrix} & \text{HS} & \text{BS} & \text{ED} & \text{OT} \\ \text{HS} & \left( \begin{matrix} 3.122 & 1.286 & 2.135 & 1.465 \\ 2.784 & 5.005 & 2.936 & 7.854 \\ 4.372 & 5.028 & 5.083 & 4.388 \\ 3.699 & 3.219 & 2.611 & 3.531 \end{matrix} \right) \\ \text{BS} & \\ \text{ED} & \\ \text{OT} & \end{matrix}$$

Information of  $M_{ij}$  besides indicating the relative importance of individual loan type also gives us on the degree of difficulty of switching among different types of loan. High value of  $M_{ij}$  indicates switching is often made. In the context of demand theory high  $M_{ij}$  value indicates an elastic demand. Like wise low  $M_{ij}$  value indicates an inelastic demand. Thus the relative importance of loan type could be deducted from this information. The diagonal values of matrix  $M$  gives us the relative importance: housing loan with  $M_{HS,HS} = 3.122$ , followed by other loan with  $M_{OT,OT} = 3.531$ ,

business loan with  $M_{BS,BS} = 5.005$  and education loan with  $M_{ED,ED} = 5.083$ . Moreover if a row analysis on the  $M$  matrix is made, one would observe that the time to switch from business and education loan to housing loan is relatively shorter than switching to other types of loan, making housing loan the best preferred loan among the consumers. Thus both information on steady state distribution and the first passage time lead us to the same conclusion on the relative importance of the various type of loan.

**Forecast on Loan Disbursement Proportion:** One of the advantages of using a Markov model in analyzing the loan portfolio mix, besides understanding its basic characteristic is its ability to make forecast on the proportion. In this study we shall forecast the loan disbursement proportion for the year 2002. Needless to say that the accuracy of a forecasting technique certainly depends on the availability of past and current data, and as such forecasting work is always a dynamic process. Forecasting the monthly proportion of all loan types is again based on the following relation  $\hat{X}(T+t) = X(T)P^t$  where  $\hat{X}(T+t)$  and  $X(T)$  are vector of forecast proportion for  $\tau$  period ahead and  $X(T)$  actual proportion for December 2001 respectively.

For example  $\hat{X}(25)$  is a vector of forecast proportion for January 2002. Table 4 gives the vector of monthly forecast proportion for the year 2002.

The forecast proportion for the housing loan is at 41.22% for January 2002 while the proportion for business, education and other loans is at 16.49, 16.12 and 26.17% respectively.

Proportion for housing loan is forecasted to drop at a level of 34.66% in February 2002 and finally settled down at 32.05% in July 2002 onwards.

Table 4: Forecast of Monthly Loan Proportion for the Year 2002

t	Month	Loan Type Housing	Business	Education	Others
1	January	0.4122	0.1649	0.1612	0.2617
2	February	0.3466	0.1885	0.1846	0.2803
3	March	0.3292	0.1961	0.1932	0.2815
4	April	0.3232	0.1985	0.1958	0.2825
5	May	0.3213	0.1993	0.1967	0.2827
6	June	0.3207	0.1996	0.1970	0.2827
7	July	0.3205	0.1997	0.1971	0.2827
8	August	0.3205	0.1997	0.1971	0.2827
9	September	0.3205	0.1997	0.1971	0.2827
10	October	0.3205	0.1997	0.1971	0.2827
11	November	0.3205	0.1997	0.1971	0.2827
12	December	0.3205	0.1997	0.1971	0.2827



For the business, education and other loan types, the trend for the forecast proportion is marginally an upward trend. Business loan proportion for January 2002 is forecasted at 16.49% and increased to 19.97% in July 2002. For education and other loan, the corresponding forecast is at 16.12% increased to 19.71 and 26.17% increased to 28.27% respectively.

One obvious observation is that after July 2002, the forecasted proportions for individual loan remains the same. This phenomenon is to be expected as Markov Chain model is a short term forecasting model.

The forecast values discussed above give the policy maker an indication on the average proportion of different types of loan.

In practice forecasts have to be updated as current data are available, and it is recommended that at the beginning of a month forecasts could be made when the previous month data are known.

This will further improve the accuracy of the forecasts. Moreover if forecasts on the total allocation for retail banking loan is available, one would easily compute the individual loan allocation using the updated proportion forecasts.

The major findings of this study are as follows:

1. The implicit characteristic of the disbursement process derived from the transition probability matrix shows that loan switching is possible in the retail banking unit. The existence of non absorbing loan further indicates that the aggregate loan disbursement data is the best proxy of the individual movement of loan disbursement among its type. Non zero probability values of switching from any loan type to business loan indicate that business loan allocation is not fully utilized. Thus signifying that business loan is of less important to retail banking.
2. The steady state distribution of the loan disbursement process shows that optimal loan portfolio mix is as follows: housing loan constitutes 32 % of the retail banking unit. This proportion is a little higher than the bank's targeted proportion of 30 %. This is followed by other loan 28.3 %, business loan 20.0 % and education loan 19.7 %. This gives the information on the relative importance of the various loan in that order.
3. One of the additional advantage of using Markov Chain model in studying the loan allocation problem besides giving the probability of loan switching is the ability of model to dispense information on the degree of difficulty in making loan switching. This is possible through the mean first passage time ( $M_{ij}$ ). It had been established in previous demand studies that high value of  $M_{ii}$

indicates an elastic demand and likewise low value of  $M_{ii}$  indicates an inelastic demand. Since loan disbursements can be considered as demand for loan, then the analyses on  $M_{ij}$  values indicate that the demand for housing loan and other loan are relatively inelastic while business and education loans demand are relatively elastic. From the economic theory point of view, the bank may increase the interest rate on the housing and other loans and yet still be able to sustain demand. Unlike the other two loans, any increase in interest rate will cause substantial reductions in loan demand.

4. The rate of convergence to the equilibrium state is the measurement of how fast the process reach its equilibrium state. This information could be obtained through the eigen values of the transition probability matrix. Alternatively, one would analyze the behavior of the loan proportion forecasts as given in Table 3. It is observed that the forecast proportions beginning July 2002 for all loan types are the same. This indicates that the loan disbursement process reaches the equilibrium state in a shorter period of time signifying matured loan demand process. One would view matured loan demand process as the ability of the bank to declassify the loan disbursement according to its types. Thus, shorter period means that the bank is able to declassify it without much difficulty.
5. This study further stimulates the expected income on interest by using the Markov proportions and the forecast on the value of loans in each type. It had been proved that loan allocation using Markov proportions yields higher expected income on interest and considered superior to the existing policy.

## CONCLUSION

The preceding discussions can be concluded in the following lines that among the four types of loans, housing loan is expected to constitute 32.0 % of the retail banking. This is followed by other type 28.3 %, business loan 20.0 % and education loan 19.7 %. In addition, information on the mean passage time confirms that the order of loan importance should be in that order of sequence only. Finally, in order to rationalize these findings and to show that loan allocation using Markov Chain model yields higher expected income the interest estimation process was carried out and compared with the actual interest to prove the superiority. From the results it was concluded that this model is superior to the other non documented model which was practised by the bank.

## REFERENCES

1. Bank Negara Malaysia, 1999. The Central Bank and The Financial System In Malaysia –a Decade of Change 1989-1999. Kuala Lumpur: Bank Negara Malaysia.
2. Berger, A.N., and G.F.Udel, 1996. Universal Banking and the Future of Small Business Lending. In: Sauders, A., Walter, I., (eds), Universal Banking: Financial System Design Reconsidered. Irwin, Chicago, IL, pp. 558-627.
3. Lockett, D.G., 1984. Money & Banking (3<sup>rd</sup>. Edition). New York: McGraw Hill.
4. Bank Negara Malaysia, 2001. Annual Report 2000.
5. Ahmad Kaleem, 2000. Modelling Monetary Stability Under Dual Banking System: The Case of Malaysia. Int. J. Islamic Financial Services, Vol. 2, No. 1.
6. Kosubud, H. and H. Stokes, 1980. OPEC Short Term Market Share Behaviour : Implication Theories and Facts., Energy Economics, April.
7. Yushkevich, A., 2001. Optimal Switching Problems for Countable Markov Chains: Average Reward Criterion. Mathematical Method Operations Res. 53: 1-24.
8. Bank Negara Malaysia, 2001. The Masterplan: Building A Secure Future. Kuala Lumpur.
9. Bank Negara Malaysia: Annual Reports (Various Issues)
10. Gorton, G., and J. Kahn, 1993. The Design of Bank Loan Contracts, Collateral and Renegotiation, National Bureau of Economic Research, Working Paper.
11. Banking and Financial Institution Act (BAFIA), 1989. Govt. of Malaysia
12. Bank Negara Malaysia, 2001. Annual Report 2000, Bumiputra-Commerce Bank, Economic Update, Economic Research Services, (Various Volumes) 1999-2002 and Commerce Asset-Holding Berhad : Annual Report 1999-2000
13. Berger, A.N., A. Saunders, J.M. Scalise and G.F. Udell, 1998. The Effects of Bank Mergers and Acquisitions on Small Business Lending. J. Financial Economics, 50: 187-229
14. Kallberg, J.G. and A. Saunders, 1995. Markov Chain Approaches to the Analysis of Payment Behavior of Retail Credit Customers, Springer, New York
15. Kaufmann, S., 2000. Measuring Business Cycles with a Dynamic Markov Switching Factor Model: An Assessment Using Bayesian Simulation Methods. Econometric J. 3: 39-65.
16. Mohd. Yusof, A., 1998. A Survey of Management Science and Operations Research Techniques In Malaysia. Annual Review of Operations Research and Management Science. 1: 14-32.
17. Kholodilin, K., 2001. Latent Leading and Coincident Factors Model with Markov – Switching Dynamics. Economic Bulletin. 3: 1 – 13.
18. Lee.C., G. Judge and T. Takayama, 1965. On Estimating the Transition Probabilities of a Markov Process, J. Farm Economics, Vol.47.
19. Mandansky, A., 1959. Least Squares Estimation in Finite Markov Processes, Psychometrika, Vol. 24.
20. Bardaie, M.Z. and A. Salam, 1981. A Stochastic Model of Daily Rainfall for University Pertanian Malaysia. Pertanika,.4: 1-9.
21. Honohan, P., 1999. A Model of Bank Contagion Through Lending, International Review of Economics and Finance, 8: 147-163
22. Mahmood, R., 2000. Influence of Heuristics in Bank Manager's Lending Decisions to Small Business, Bankers J. Malaysia, 116: 34-37
23. Mohd. Yusof, A., 1982. Forecasting Demand Share of Petroleum Products, Jurnal Ekonomi Malaysia, 6:1-14