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A Bipolar Cockcroft-Walton Voltage Multiplier for Gas Lasers

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Abstract: A bipolar Cockcroft-Walton Voltage Multiplier (CWVM) is proposed as an attractive alternative to the symmetrical Cockcroft-Walton voltage multiplier for continuous wave gas lasers (e.g. carbon dioxide gas laser). The proposed CWVM formed by combining positive and negative voltage multipliers consisting of equal number of stages and driving in parallel by an ac voltage source. The proposed voltage multiplier needs only one ac power source, therefore there was no need of center taped high voltage transformer, unlike symmetrical voltage multiplier which require center-tape transformer. It possess inherit ability of cancellation of fundamental and higher order odd harmonics of ripple components, unlike the symmetrical CWVM which may generate odd harmonic of ripple in case of any asymmetry of driving voltage. In addition to this the proposed voltage multiplier has faster transient rise and less voltage drop as compared symmetrical CWVM. The experimental and simulation results are presented to show the effectiveness of proposed voltage multiplier.

Key words: Voltage multiplier, asymmetry of driving voltage, fundamental harmonics, voltage ripple

INTRODUCTION

Although Cockcroft-Walton voltage multiplier (CWVM) circuit was developed long time ago in 1932, it is still widely used in many high-voltage low-current applications such as lasers, accelerators, ultra-high voltage electron microscopes, and x-ray power generators[1-6] etc. The original CWVM circuit was of half-wave (asymmetrical) type and it has large output voltage ripple and voltage drop[1-5]. A number of modifications of the original CWVM circuit have been proposed and applied to reduce steady state voltage drop and voltage ripple[7-8]. A symmetrical CWVM which is an improved form of original CWVM is presently more popular and is widely used in most of the above mentioned application. It has significantly smaller output voltage ripple and voltage drop as compared to original CWVM[9-11]. This is because the symmetrical structure of symmetrical CWVM cancels out the fundamental harmonic of ripples caused by driving voltage and stray capacitance. Thus the load generated second order harmonic is the major ripple component in the dc output of symmetrical CWVM. The second and higher order even harmonics of ripples are proportional to load current and can be minimized by choosing larger size of smoothing column capacitors[9-13]. However the circuit asymmetry, especially the asymmetry of the driving voltage may deteriorate the cancellation effect and give rise to generation of fundamental and higher order odd harmonic of ripples[12-13]. The fundamental harmonic of ripples increases with the increase in the asymmetry of driving voltage and in case of low load current it may dominate over the second harmonic. This is due to reason that at lower load current the peak to peak value of load generated second harmonic of ripples would be smaller than the fundamental harmonic. The fundamental harmonic component was found to be dominant in some lower load current application such as in some electron microscopes and accelerators where symmetrical CWVM is used as high voltage generator[12-13]. Similarly the fundamental harmonics also effects the quality of output laser beam of continuous wave carbon dioxide gas laser.

To overcome this problem of asymmetry of driving voltage and to get output voltage free from fundamental and higher order odd harmonics we have proposed a bipolar CWVM. The proposed CWVM has intrinsic ability to cancel the fundamental and odd harmonic of ripples caused by driving voltage. In addition to this the proposed voltage multiplier has many advantages over the symmetrical CWVM. These include smaller size, light weight, less component counts, easier implementation, faster transient response and smaller voltage drop as compared to symmetrical CWVM.

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Circuit Description and Principle of Operation: Figure 1 shows the circuit diagram of proposed n-stage bipolar Cockcroft-Walton voltage multiplier. It consists of a p-stage positive and q-stage negative Cockcroft-Walton voltage multipliers, where as $p = q = n/2$ and $p + q = n$, here n is the total number of stages of proposed bipolar CWVM. The two voltage multipliers are connected in parallel to the secondary of high voltage transformer whose primary is driven by a voltage source $V(t) = V\sin(\omega t)$. The load is connected between the output terminal of positive CWVM and negative CWVM as shown in the Fig. 1. Therefore voltage $V_\circ$ across the load is sum of the output voltage of positive voltage multiplier (V_{ab}) and negative voltage multiplier (V_{bc}). The operation of proposed bipolar CWVM can be explained as; when the input voltage V_S swings positively, the diodes $D_1^b, D_2^b ----D_p^b$ of positive voltage multiplier and the diodes $D_1^a, D_2^a ----D_q^a$ of negative voltage multiplier turns on. Thus during this interval the smoothing column $(C_1, C_2, ----C_p)$ of positive voltage multiplier is charged and smoothing column is discharged. Similarly as the input ac voltage swings negatively then the diodes $D_1^a, D_2^a ----D_p^a$ of positive voltage multiplier and diodes $D_1^b, D_2^b ----D_q^b$ of negative voltage multiplier turns ON. During this interval smoothing column of positive voltage multiplier is discharged and that of positive voltage multiplier is charged. Now as the smoothing columns of two voltage multiplier charges in alternative cycles therefore the frequency of ripple in the dc output $V_\circ$ is twice the input ac voltage frequency.

Output voltage ripple: There are two types of ripple components that exist in the individual output of both positive and negative voltage multiplier. The first type of ripple component is of sinusoidal shape and is produced by currents which circulate in the series-shunt capacitors. This type of ripple component is completely cancelled as the output of positive and negative voltage multiplier are added together. For analytical proof let us suppose that C_S is the capacitance across each rectifier and all capacitors used in voltage multiplier are equal (i.e. $C_p's = C_q's = C$). The ripple in the dc output of positive voltage multiplier due to circulating currents in the series-shunt capacitors is given by [12];

$$\delta V_F^{ab} = \frac{V_S\sqrt{C_S/C}\,\sinh^2(p\sqrt{C_S/C})}{\cosh(2p\sqrt{C_S/C})\sinh(\sqrt{C_S/C})} \qquad (1)$$

Similarly the ripple due to circulating currents in the dc output of negative voltage multiplier is given by;

$$\delta V_F^{bc} = -\frac{V_S\sqrt{C_S/C}\,\sinh^2(q\sqrt{C_S/C})}{\cosh(2q\sqrt{C_S/C})\sinh(\sqrt{C_S/C})} \qquad (2)$$

Here negative sign is due to the reason that there exists a phase difference of 180° between the output ripple components of positive and negative voltage multiplier. Now summing equation (1) and (2) for finding total ripple harmonic due to circulating currents in the dc output $V_\circ$ of the proposed bipolar voltage multiplier we have;

$$\delta V_F = \frac{V_S\sqrt{C_S/C}\,\sinh^2(p\sqrt{C_S/C})}{\cosh(2p\sqrt{C_S/C})\sinh(\sqrt{C_S/C})} - \frac{V_S\sqrt{C_S/C}\,\sinh^2(q\sqrt{C_S/C})}{\cosh(2q\sqrt{C_S/C})\sinh(\sqrt{C_S/C})}$$

Putting $p = q = n/2$ we have;

$$\delta V_F = \frac{V_S\sqrt{C_S/C}\,\sinh^2(n/2\sqrt{C_S/C})}{\cosh(n\sqrt{C_S/C})\sinh(\sqrt{C_S/C})} - \frac{V_S\sqrt{C_S/C}\,\sinh^2(n/2\sqrt{C_S/C})}{\cosh(n\sqrt{C_S/C})\sinh(\sqrt{C_S/C})} = 0 \qquad (3)$$

This proves that ripple component due circulating currents are cancelled as the output of positive and negative voltage multipliers are added together. The cancellation of this kind of ripple component occurs because of the phase difference as the outputs of positive and negative voltage multipliers are added together. Thus the output of proposed bipolar CWVM is free from this kind of ripple component. Therefore the ripple due to circulating currents are completely absent in the dc output of proposed voltage multiplier.

The second type of ripple component is due to periodic charging and discharging of smoothing column capacitors. The smoothing column capacitors of both positive and negative voltage multiplier are discharged by load current and are recharged to peak value once every cycle. However as the charging of smoothing column of positive voltage multiplier occurs when input

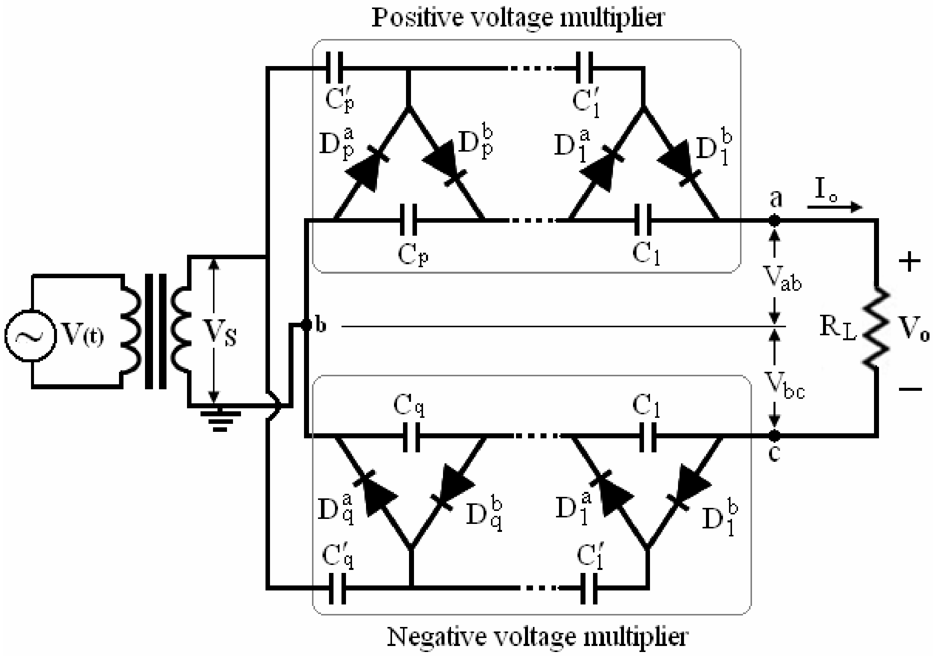

Fig. 1: Proposed n-stage bipolar Cockcroft-Walton voltage multiplier

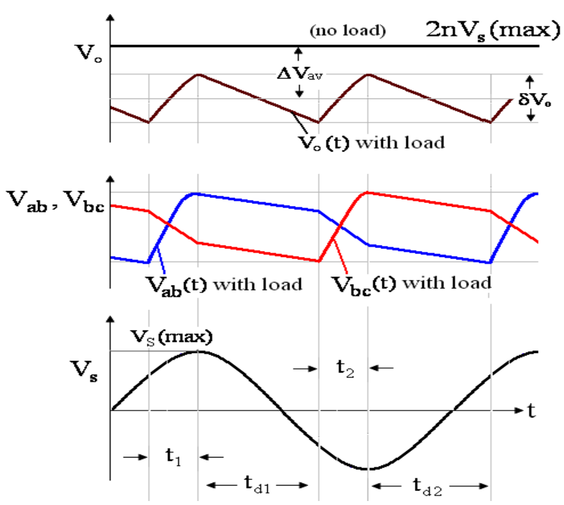

Fig. 2: Typical steady state waveform of proposed bipolar CWVM

voltage V_S swings positively and that of negative voltage multiplier when the input voltage V_S swings negatively as shown in the key steady state waveform of the Fig. 2. Therefore the ripple in the resulting output V_o of proposed voltage multiplier is of second order of the drive signal frequency. As a result the fundamental and higher order odd harmonic components of load current generated voltage ripple are also cancelled and the dc output of proposed bipolar voltage multiplier contains only second and higher order even harmonics with second harmonic being the most significant.

To estimate the total output ripple voltage let us suppose that $Q = I \cdot T = I/f$ is the charge transferred to load per cycle. This charge is supplied by the series connected capacitors of smoothing column $(C_1, C_2, ----C_p)$ and $(C_1, C_2, ----C_q)$ of positive and negative voltage multipliers. The smoothing column $(C_1, C_2, ----C_p)$ is charged to peak value by respective oscillating columns during time interval t_1 and the smoothing column $(C_1, C_2, ----C_q)$ is charged to peak value during time interval t_2. Now to calculate the peak to peak voltage ripple let us consider the charging time interval t_1. In this interval the capacitors of smoothing column of positive voltage multiplier are charged. The change or ripple δV_{ab} in the dc output of positive voltage multiplier due to charging of smoothing column capacitor after being discharged by load current is given by;

$$\delta V_{ab}=\left[\frac{pQ}{C_p}+\frac{(p-1)Q}{C_{p-1}}+\cdots\cdots+\frac{2Q}{C_2}+\frac{Q}{C_1}\right]$$

Now as in time interval t_1 the capacitors of smoothing column of negative voltage multiplier transfer charge to oscillating column capacitors, therefore the output voltage of the negative voltage multiplier are reduced in the interval. The ripple δV_{bc} that results due to loss of charge in time interval t_1 by smoothing column capacitors of negative voltage multiplier is give by;

$$\delta V_{bc}=-\left[\frac{(q-1)Q}{C_q}+\frac{(q-2)Q}{C_{q-1}}+\cdots\cdots+\frac{Q}{C_2}+\frac{0}{C_1}\right]$$

The negative sign here indicates that the change in output voltage is negative. Now summing δV_{ab} and δV_{bc} for total ripple δV in the dc output $V_\circ$ we have;

$$\delta V=\left[\frac{pQ}{C_p}+\frac{(p-1)Q}{C_{p-1}}+\cdots\cdots+\frac{2Q}{C_2}+\frac{Q}{C_1}\right]-\left[\frac{(q-1)Q}{C_q}+\frac{(q-2)Q}{C_{q-1}}+\cdots\cdots+\frac{Q}{C_2}+\frac{0}{C_1}\right]$$

As $p=q=n/2$ and assuming, $C_p's=C_q's=C$;

$$\delta V=\left[\frac{nQ}{2C}+\frac{(n/2-1)Q}{2C}+\cdots\cdots+\frac{2Q}{2C}+\frac{Q}{2C}\right]-\left[\frac{(n/2-1)Q}{2C}+\frac{(n/2-2)Q}{2C}+\cdots\cdots+\frac{Q}{2C}+\frac{0}{2C}\right]$$

Or

$$\delta V=\frac{1}{2}\left[\frac{nQ}{C}+\frac{(n/2-1)Q}{C}+\cdots\cdots+\frac{2Q}{C}+\frac{Q}{C}\right]-\left[\frac{(n/2-1)Q}{C}+\frac{(n/2-2)Q}{C}+\cdots\cdots+\frac{Q}{C}+\frac{0}{C}\right]$$

After canceling similar terms we have;

$$\delta V=\frac{Q}{C}\frac{n}{2}=\frac{I}{fC}\frac{n}{2} \qquad (4)$$

This result is similar to that of symmetrical Cockcroft-Walton voltage multiplier. So the peak to peak output voltage ripple of proposed bipolar voltage multiplier is equal to symmetrical CWVM. However as there is only single driving ac voltage, therefore there is no fundamental ripple component due to asymmetry of driving voltage. This is the advantage over symmetrical CWVM.

Output voltage drop: The output voltage drop of proposed bipolar CWVM is given by;

$$\Delta V_{total}=\Delta V_{ab}+\Delta V_{bc} \qquad (5)$$

Where, ΔV_{bc} and ΔV_{bc} is the voltage drop on load of positive and negative voltage multiplier respectively and is given by;

$$\Delta V_{ab}=\frac{I_\circ}{fC}\left(\frac{2}{3}p^3+\frac{p^2}{2}-\frac{p}{6}\right) \qquad (6)$$

and

$$\Delta V_{bc}=\frac{I_\circ}{fC}\left(\frac{2}{3}q^3+\frac{q^2}{2}-\frac{q}{6}\right) \qquad (7)$$

Substituting $p=q=n/2$ in equation (6) & (7), we get;

$$\Delta V_{ab}=\frac{I_\circ}{fC}\left(\frac{n^3}{12}+\frac{n^2}{8}-\frac{n}{12}\right) \qquad (8)$$

$$\Delta V_{bc}=\frac{I_\circ}{fC}\left(\frac{n^3}{12}+\frac{n^2}{8}-\frac{n}{12}\right) \qquad (9)$$

Putting equations (8) and (9) in (5), we have;

$$\Delta V_{total}=\frac{I_\circ}{fC}\left(\frac{n^3}{6}+\frac{n^2}{4}-\frac{n}{6}\right) \qquad (10)$$

Equation (10) shows that proposed bipolar CWVM has slightly smaller current dependent voltage drop, therefore it has better regulation as compared to symmetrical CWVM.

Average output voltage: The average output voltage is calculated as;

$$V_\circ(av)=(V_{ab}+V_{bc})-\Delta V_{total}-\frac{\delta V}{2}$$

$$V_\circ(av)=(nV_S+nV_S)-\frac{I_\circ}{fC}\left(\frac{n^3}{6}+\frac{n^2}{4}-\frac{n}{6}\right)-\frac{I_\circ}{fC}\frac{n}{4}$$

$$V_\circ(av)=2nV_S-\frac{I_\circ}{fC}\left(\frac{n^3}{6}+\frac{n^2}{4}+\frac{n}{12}\right) \qquad (11)$$

Equation (10) and (11) gives the voltage drop on load and average output voltage of proposed bipolar CWVM respectively. Table 1 compares the calculated voltage ripple δV, average voltage-drop ΔV_{av} and mean output voltage $V_\circ(av)$ of the proposed bipolar voltage multiplier with the corresponding values for the conventional asymmetrical CWVM and symmetrical CWVM. The comparison shows that the proposed CWVM has smaller voltage drop and voltage ripple than the conventional half-wave CWVM. It has slightly

Table 1: Comparison between various voltage multiplier circuits

	$V_o(av)$	(ΔV_{av})	(δV)
Asymmetrical CWVM	$2nV_S(\max) - \frac{I_o}{fC}\left[\frac{2n^3}{3} + \frac{n^2}{2} + \frac{n}{3}\right]$	$\frac{I_o}{fC}\cdot\left[\frac{2n^3}{3} + \frac{n^2}{2} + \frac{n}{3}\right]$	$\frac{I_o}{fC}\cdot\frac{n(n+1)}{2}$
Symmetrical CWVM	$2nV_S(\max) - \frac{I_o}{fC}\left[\frac{n^3}{6} + \frac{n^2}{4} + \frac{n}{3}\right]$	$\frac{I_o}{fC}\cdot\left[\frac{n^3}{6} + \frac{n^2}{4} + \frac{n}{3}\right]$	$\frac{I_o}{fC}\cdot\frac{n}{2}$
Proposed bipolar CWVM	$2nV_S(\max) - \frac{I_o}{fC}\left[\frac{n^3}{6} + \frac{n^2}{4} + \frac{n}{12}\right]$	$\frac{I_o}{fC}\cdot\left[\frac{n^3}{6} + \frac{n^2}{4} + \frac{n}{12}\right]$	$\frac{I_o}{fC}\cdot\frac{n}{2}$

smaller voltage drop than symmetrical CWVM and equal voltage ripple. In addition to above advantage the proposed bipolar voltage multiplier also has faster transient response. This is because the two voltage multipliers which consist of half the total number of stages are driven in parallel. The number of stage connected in series is reduced to half and therefore the time required to traverse the series connected capacitors is reduced. As transient rise time of the voltage multiplier circuit depends upon the number of stages connected in series.

Features of Proposed Bipolar CWVM: The proposed bipolar voltage multiplier circuit has many advantages over the symmetrical voltage multiplier such as:

1. it require only one secondary winding of high voltage transformer therefore the size and weight of high voltage transformer is reduced and the construction becomes easier
2. the number of capacitor and diodes required are approximately half to that of symmetrical voltage multiplier. For example to implement a symmetrical CWVM we need 16 diodes and 12 capacitors, where as we need only 8 diodes and 8 capacitors to implement a 4-stage proposed bipolar CWVM.
3. it has faster transient response as compared to symmetrical voltage multiplier
4. the fundamental and higher order odd harmonic of ripple is not present in the output voltage unlike symmetrical whichmay generate fundamental harmonic due to any asymmetry of driving voltage
5. the fundamental and higher order odd harmonic of ripple is not present in the output voltage unlike

RESULTS AND DISCUSSION

The effectiveness of the proposed technique is verified on laboratory prototype of Fig. 1. The specifications of the experimental prototypes are as follows: the number of voltage multiplier stages: $n = 4$, size of voltage multiplier capacitors: $C_R = C_L = C = 10nF$, output load resistance: $R_L = 68k\Omega$, operating frequency: $f = 39kHz$, input driving voltage: $V_S(\max) = 40V$. In order to compare the performance a laboratory prototype of symmetrical CWVM was also built with same specifications.

Fig. 3 shows the simulated input and output voltage waveforms of proposed bipolar CWVM. These simulated waveforms are exactly similar to key steady state waveforms of Fig. 2. The ripples contained in the individual dc outputs V_{ab} of positive CWVM and V_{bc} of negative CWVM have same frequency as that of the input driving voltage. However the frequency of the ripples contained in the dc output V_o of proposed bipolar CWVM is of the second order of the input driving voltage frequency. This confirms that the fundamental ripple harmonics is absent in the dc output of proposed bipolar CWVM.

Fig. 4 and 5 shows the experimental and simulated output voltage waveform of proposed bipolar CWVM and symmetrical CWVM during start up process for equal load resistance respectively. The transient rise time of output voltage of proposed bipolar CWVM is small as compared to symmetrical CWVM. The output voltage of proposed bipolar CWVM is also larger than symmetrical CWVM. This proves that the proposed bipolar CWVM has faster dynamic response and smaller current dependent voltage drop as compared to symmetrical CWVM. Figure 6 and 7 shows the experimental and simulated waveforms of output voltage ripple of both proposed bipolar voltage multiplier and symmetrical voltage multiplier. Both experimental and simulation results are well in agreement with each other. The frequency of output voltage ripple of proposed bipolar CWVM and that of the symmetrical CWVM is of the second order of the driving signal frequency. The peak to peak value of output ripple of proposed bipolar CWVM is slightly larger than the symmetrical CWVM. This is due to reason that the output voltage of proposed bipolar CWVM is larger than the symmetrical CWVM.

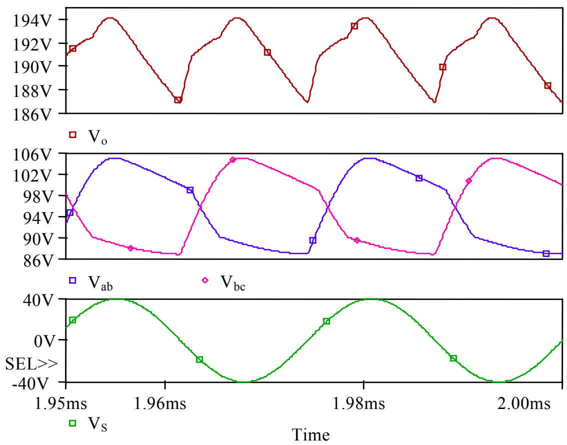

Fig. 3: Simulated input and output voltage waveforms of proposed bipolar CWVM in steady state

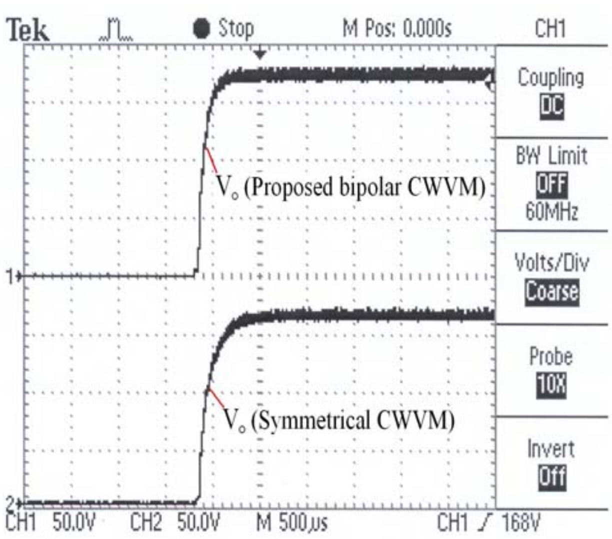

Fig. 4: Experimental output voltage waveforms of proposed CWVM and symmetrical CWVM during start-up process

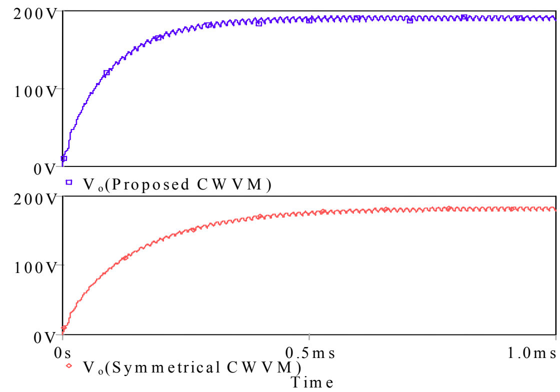

Fig. 5: Simulated output voltage waveforms of proposed CWVM and symmetrical CWVM during start-up process

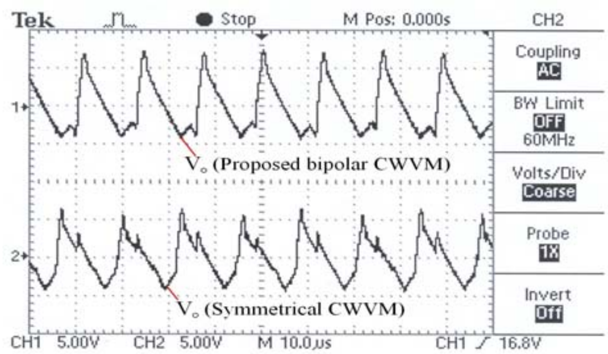

Fig. 6: Experimental steady state waveforms of output voltage of proposed bipolar CWVM and symmetrical CWVM

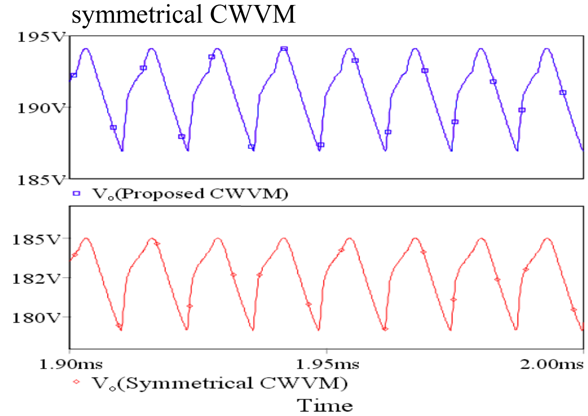

Fig. 7: Simulated steady state waveforms of output voltage of proposed bipolar and symmetrical CWVM

CONCLUSION

A bipolar CWVM as an attractive alternative to the symmetrical Cockcroft-Walton voltage multiplier for continuous wave gas lasers has been proposed in this paper. The proposed bipolar CWVM has been shown to have superior performance over symmetrical CWVM. It require only one ac power source, therefore there is no need of center taped high voltage transformer, unlike symmetrical voltage multiplier which require center-tape transformer. It possess inherit ability of cancellation of fundamental and higher order odd harmonics of ripple components, unlike the symmetrical CWVM which may generate odd harmonic of ripple in case of any asymmetry of driving voltage. In addition to this the proposed voltage multiplier has faster transient rise and less voltage drop as compared symmetrical CWVM. The experimental and simulation results are presented to show the effectiveness of proposed voltage multiplier.

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