Nonlinear Whitham-Broer-Kaup Wave Equation in an Analytical Solution

S.A. Zahedi, M.Vaezi and N. Tolou
Department of Mechanical Engineering,
Mazandaran University, Babol, P.O. Box 484, Iran

Abstract: This study presented a new approach for the analysis of a nonlinear Whitham-Broer-Kaup equation dealing with propagation of shallow water waves with different dispersion relations. The analysis was based on a kind of analytical method, called Variational Iteration Method (VIM). To illustrate the capability of the approach, some numerical examples were given and the propagation and the error of solutions were shown in comparison to those of exact solution. In clear conclusion, the approach was efficient and capable to obtain the analytical approximate solution of this set of wave equations while these solutions could straightforwardly show some facts of the described process deeply such as the propagation. This method can be easily extended to other nonlinear wave equations and so can be found widely applicable in this field of science.

Key words: Variational iteration method (VIM), propagation of wave equations, whitham-broer-kaup equation

INTRODUCTION

Most of scientific problems and phenomena occur nonlinearly. Except a limited number of these problems, most of them do not have precise analytical solutions so that these have to be solved using other methods. Many different new methods have recently presented; for example, Variational Iteration Method (VIM)[1-3], Homotopy Perturbation Method (HPM)[4-8] Homotopy Analysis Method[9] and Exp function[10].

In this article, VIM is used to solve nonlinear equations of Whitham-Broer-Kaup with the initial conditions u(x, 0) = F(x), v(x, 0) = G(x) for the initial time.

$$u_t + uu_x + v_x + \beta u_{xx} = 0 \qquad (1a)$$

$$v_t + (uv)_x + \alpha u_{xxx} - \beta v_{xx} = 0 \qquad (1b)$$

If $\alpha = 0$, $\beta \neq 0$, system (1) becomes classical long wave equation that describe shallow water wave with dispersive[11]. If $\alpha = 1$, $\beta = 0$, system (1) becomes variant Boussinesq equation. In this research we will focus on finding analytical approximate and exact traveling wave solution of the system (1) using the variational iteration method.

MATERIALS AND METHODS

Fundamental of variational iteration method: To clarify the basic ideas of VIM, we consider the following differential equation[1-3]:

$$Lu + Nu = g(t) \qquad (2)$$

where, L is a linear operator, N a nonlinear operator and g(t) an inhomogeneous term.

According to VIM, we can write down a correction functional as follows[1-3]:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda \left(Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau) \right) d\tau \qquad (3)$$

where, λ is a general Lagrangian multiplier which can be identified optimally via the variational theory. The subscript n indicates the 9th approximation and $\tilde{u}_n$ is considered as a restricted variation $\delta \tilde{u}_n = 0$.

Implementation of Variational Iteration Method: We first consider the application of VIM to WBK Eq. 1 subject to the following initial conditions[11]:

$$u(x,0) = \omega - 2Bk \coth(k\xi) \qquad (4a)$$

$$v(x,0) = -2B(B+\beta)k^2 \operatorname{csch}^2(k\xi) \qquad (4b)$$

Corresponding Author: S.A. Zahedi, Department of Mechanical Engineering, Mazandaran University, Babol, P.O. Box 484, Iran

where, $B = \sqrt{\alpha + \beta^2}$ and $\xi = x + x_0$ and x_0, k, ω are arbitrary constants.

Its correction variational function in x and t can be expressed respectively as follows:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda_1(\tau) \left\{ \begin{matrix} \dot{u}_n + u_n u'_n \\ + v'_n + \beta u''_n \end{matrix} \right\} d\tau \qquad (5)$$

$$v_{n+1}(x,t) = v_n(x,t) + \int_0^t \lambda_2(\tau) \left\{ \begin{matrix} \dot{v}_n + (u_n v_n)' \\ + \alpha u'''_n - \beta v''_n \end{matrix} \right\} d\tau \qquad (6)$$

where, prime indicates a differential with respect to x and dot denotes a differential with respect to t, λ_1 and λ_2 are general Lagrangian multipliers.

After some calculations, we obtain the following stationary conditions:

$$\lambda'_1(\tau) = 0 \qquad (7a)$$

$$1 + \lambda_1(\tau)|_{\tau=t} = 0 \qquad (7b)$$

$$\lambda'_2(\tau) = 0 \qquad (8a)$$

$$1 + \lambda_2(\tau)|_{\tau=t} = 0 \qquad (8b)$$

Equation 7a and 8a are called Lagrange-Euler equations and Eq. 7b and 8b are natural boundary conditions.

The Lagrange multipliers can therefore, be identified as $\lambda_1 = \lambda_2 = -1$ and the variational iteration formula is obtained in the form of:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left\{ \dot{u}_n + u_n u'_n + v'_n + \beta u''_n \right\} d\tau \qquad (9)$$

$$v_{n+1}(x,t) = v_n(x,t) - \int_0^t \left\{ \dot{v}_n + (u_n v_n)' + \alpha u'''_n - \beta v''_n \right\} d\tau \qquad (10)$$

We start with the initial approximation of u (x, 0) and v (x, 0) given by Eq. 4. Using the above iteration formulas 9 and 10, the other components can be obtained directly as follows:

$$u_0(x,t) = \omega - 2Bk \coth(k\xi) \qquad (11)$$

$$v_0(x,t) = -2B(B+\beta)k^2 \csc h^2(k\xi) \qquad (12)$$

$$u_1(x,t) = -\Big[-\omega \sinh(k\xi)\cosh^2(k\xi) + \omega \sinh(k\xi)$$
$$+ 2Bk \cosh^3(k\xi) - 2Bk \cosh(k\xi) +$$
$$2Bk^2 t\omega \sinh(k\xi) \Big]$$
$$/ \left\{ \Big[\cosh^2(k\xi) - 1 \Big] \sinh(k\xi) \right\} \qquad (13)$$

$$v(x,t) = 2Bk \Big[-\cosh(k\xi)B + B - \cosh(k\xi)\beta + \beta$$
$$+ 4Bk t\cosh(k\xi) + 2Bk t$$
$$- 2Bk \cosh(k\xi)t\omega \sinh(k\xi)$$
$$- 2k \cosh(k\xi)t\omega\beta \sinh(k\xi)$$
$$- 4\alpha tk \cosh(k\xi) - 2\alpha tk$$
$$- 4\beta tk \cosh(k\xi) - 2\beta tk \Big]$$
$$/ \Big[\cosh(k\xi) - 1 \Big] \qquad (14)$$

In the same manner, the rest of the components of the iteration formula can be obtained.

$$u(x,t) = u(x,t) - \left\{ \dot{u} + u u' + v' + \beta u'' \right\} d\tau \qquad (15)$$

$$v(x,t) = v(x,t) - \left\{ \dot{v} + (u v)' + \alpha u''' - \beta v'' \right\} d\tau \qquad (16)$$

Exact solution of WBK Eq. 1 is as follows:

$$u(x,t) = \omega - 2Bk \cosh\big[k(\xi - \omega t) \big] \qquad (17)$$

$$v(x,t) = -2B(B+\beta)k \csc h \big[k(\xi - \omega t) \big] \qquad (18)$$

These solutions are constructed by[12].

Special case one: As a special case, if $\alpha = 1$ and $\beta = 0$, WBK Eq. 1 can be reduced to the Modified Boussinesq (MB) Eq. 19:

$$u + uu + v = 0 \qquad (19a)$$

$$v + (uv) + u = 0 \qquad (19b)$$

Solving these equations by VIM as follows:

$$u(x,t) = u(x,t) + \lambda(\tau) \left\{ \dot{u} + u u' + v' \right\} d\tau \qquad (20)$$

$$v(x,t) = v(x,t) + \lambda(\tau) \left\{ \dot{v} + (u v)' + u'' \right\} d\tau \qquad (21)$$

We consider the initial conditions of the MB Equations[11]:

$$u(x,0) = \omega - 2k \coth(k\xi) \qquad (22a)$$

$$v(x,0) = -2k \operatorname{csch}(k\xi) \qquad (22b)$$

The Lagrange multipliers can therefore, be identified as $\lambda_1 = \lambda_2 = -1$ and the variational iteration formula is obtained in the following form:

$$u_{-}(x,t) = u(x,t) - \{\dot{u} + u\,u' + v'\}\,d\tau \qquad (23)$$

$$v_{-}(x,t) = v(x,t) - \{\dot{v} + (u\,v)' + u'''\}\,d\tau \qquad (24)$$

We are to start with an initial approximation $u(x,0)$ and $v(x,0)$ given by Eq. 22, by the above iteration formula 23 and 24. We can obtain directly the other components as follows:

$$u_{-}(x,t) = \omega - 2k\coth(k\xi) \qquad (25a)$$

$$v_{-}(x,t) = -2k \operatorname{csch}(k\xi) \qquad (25b)$$

$$u_1(x,t) = -\begin{bmatrix} -\omega\sinh(k\xi)\cosh^2(k\xi) + \omega\sinh(k\xi) \\ +2k\cosh^3(k\xi) - 2k\cosh(k\xi) + \\ 2k^2 t\omega\sinh(k\xi) \end{bmatrix} \\ \Big/ \Big[(\cosh^2(k\xi) - 1)\sinh(k\xi) \Big] \qquad (26)$$

$$v(x,t) = \frac{-2k\left[\cosh(k\xi) - 1 + 2k\cosh(k\xi)t\omega\sinh(k\xi)\right]}{(\cosh(k\xi) - 1)} \qquad (27)$$

In the same manner, the rest of the components of the iteration formula can be obtained:

$$u(x,t) = u(x,t) - \{\dot{u} + u\,u' + v'\}\,d\tau \qquad (28)$$

$$v(x,t) = v(x,t) - \{\dot{v} + (u\,v)' + u'''\}\,d\tau \qquad (29)$$

The exact solution of MB Eq. 19 already has been obtained in[12] as follows:

$$u(x,t) = \omega - 2k\coth(k(\xi - \omega t)) \qquad (30)$$
$$v(x,t) = -2k \operatorname{csch}(k(\xi - \omega t)) \qquad (31)$$

Special case two: In subjected problem, if $\alpha = 0$ and $\beta = 1/2$, WBK Eq. 1 can be reduced to the Approximate Long Wave (ALW) equation in shallow water:

$$u_{.} + uu_{.} + v_{.} + \frac{1}{2}u_{.} = 0 \qquad (32a)$$

$$v_{.} + (uv)_{.} - \frac{1}{2}v_{.} = 0 \qquad (32b)$$

Considering the application of the VIM to the ALW Eq. 32 with the initial conditions[11,12]:

$$u(x,0) = \omega - k\coth(k\xi) \qquad (33a)$$

$$v(x,0) = -k \operatorname{csch}(k\xi) \qquad (33b)$$

Its correction variational functional in *x* and *t* can be expressed respectively as follows:

$$u_{-}(x,t) = u(x,t) + \lambda(\tau)\begin{Bmatrix} \dot{u} + u\,u' + \\ v' + \frac{1}{2}u'' \end{Bmatrix}d\tau \qquad (34)$$

$$v_{-}(x,t) = v(x,t) + \lambda(\tau)\begin{Bmatrix} \dot{v} + (u\,v)' \\ -\frac{1}{2}v'' \end{Bmatrix}d\tau \qquad (35)$$

The Lagrange multipliers can therefore, be identified as $\lambda_1 = \lambda_2 = -1$ and the variational iteration formula is obtained in the form of:

$$u_{-}(x,t) = u(x,t) - \begin{Bmatrix} \dot{u} + u\,u' + \\ v' + \frac{1}{2}u'' \end{Bmatrix}d\tau \qquad (36)$$

$$v_{-}(x,t) = v(x,t) - \begin{Bmatrix} \dot{v} + (u\,v)' \\ -\frac{1}{2}v'' \end{Bmatrix}d\tau \qquad (37)$$

An initial approximation $u(x,0)$ and $v(x,0)$ given by Eq. 33, by the above iteration formula 36 and 37. We can obtain directly the other components as follows:

$$u(x,t) = \omega - k\cosh(k\xi) \qquad (38)$$

$$v_0(x,t) = -k \operatorname{csch}(k\xi) \qquad (39)$$

$$v_1(x,t) = \frac{-k^2\left(\cosh^2(k\xi) - 1 + 2k\cosh(k\xi)t\omega\sinh(k\xi)\right)}{\left(\cosh^2(k\xi) - 1\right)^2} \qquad (40)$$

In the same manner, the rest of the components of the iteration formula can be obtained.

$$u_2(x,t) = u_1(x,t) - \int_0^t \left\{ u_1 + u_1 u_1' + v_1 + \frac{1}{2}u_1'' \right\} d\tau \quad (41)$$

$$v_2(x,t) = v_1(x,t) - \int_0^t \left\{ v_1 + (u_1 v_1)' - \frac{1}{2}v_1'' \right\} d\tau \quad (42)$$

Exact solution of ALW Eq. 32 is as follows[12]:

$$u(x,t) = \omega - k\coth(d(\xi - \omega t)) \quad (43)$$

$$v(x,t) = -k^2 \csc h^2(k(\xi - \omega t)) \quad (44)$$

RESULTS AND DISCUSSION

The solution of Whitham-Broer-Kaup Wave equation is successfully investigated analytically by Variational Iteration Method (VIM). For clarity, also to illustrate some facts directly from obtained solutions, the propagation of discussed wave phenomena have been shown in Fig. 1-6. Figure 1 and 2, respectively, shows the propagation of u (x, t) and v(x, t) versus time, t and displacement, x, for the general form of wave equation (Eq. 1) when $k = 0.1$, $\omega = 0.005$, $\alpha = 1.5$, $\beta = 1.5$, $x_0 = 10$. The behavior of propagation of u(x, t) and v(x, t) for special case 1 (Eq. 19) have been shown in Fig. 3 and 4 respectively. In the same manner the results of u (x, t) and v (x, t) for special case 2 (Eq. 32)

(a)   (b)

Fig. 1: Propagation of u(x,t) for Eq. 1, VIM (a) and exact solution (b) for $k = 0.1$, $\omega = 0.005$, $\alpha = 1.5$, $\beta = 1.5$, $x_0 = 10$

are shown in Fig. 5 and 6 respectively. In all figures, (a) refers to VIM and (b) refers to the exact solution. These figures illustrate well agreement between the VIM and exact solution.

For further verification, the error of Fig. 1-6a are shown in Table 1-3 in comparison to those of exact solution. These tables clearly confirm the high accuracy of obtained solutions.
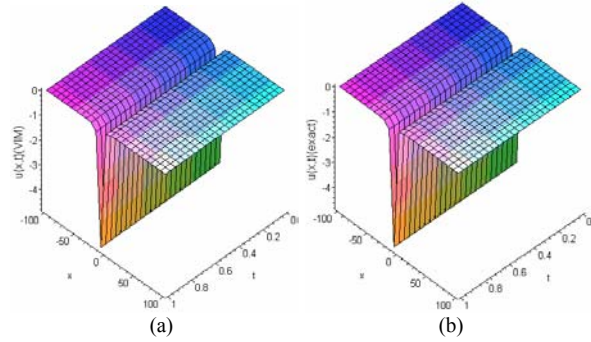

(a)   (b)

Fig. 2: Propagation of v(x,t) for Eq. 1, VIM (a) and exact solution (b) for $k = 0.1$, $\omega = 0.005$, $\alpha = 1.5$, $\beta = 1.5$, $x_0 = 10$
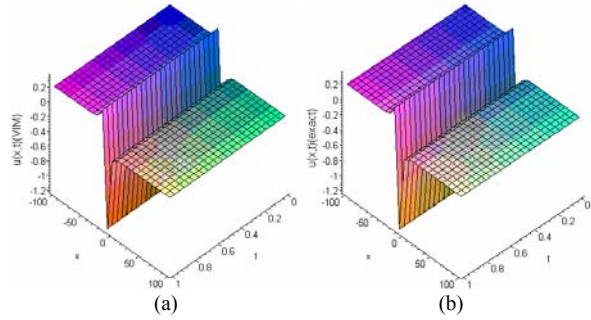

(a)   (b)

Fig. 3: Propagation of u(x,t) for Eq. 19, VIM (a) and exact solution (b) for $k = 0.1$, $\omega = 0.005$, $\alpha = 1.5$, $\beta = 0$, $x_0 = 10$

(a)   (b)

Fig. 4: Propagation of v(x,t) for Eq. 19, VIM (a) and exact solution (b) for $k = 0.1$, $\omega = 0.005$, $\alpha = 1.5$, $\beta = 0$, $x_0 = 10$

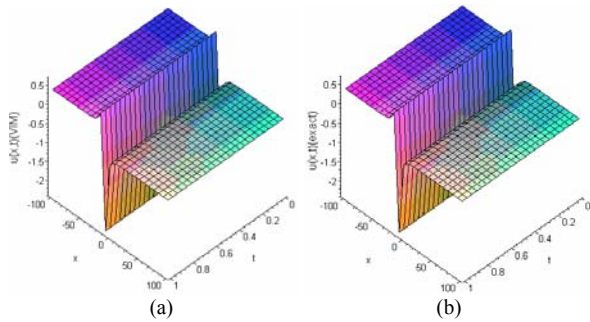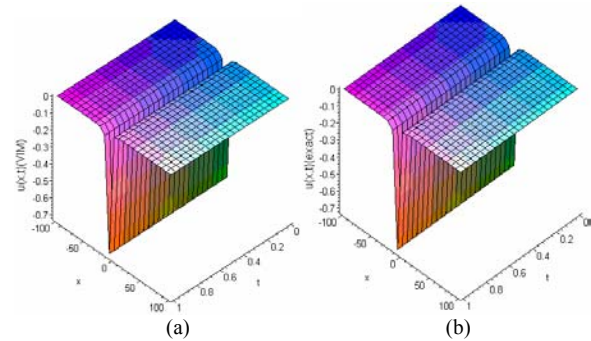Table 1: The numerical results for $\left|u_{exact}-u_{VIM}\right|=\psi$ and $\left|v_{exact}-v_{VIM}\right|=\varphi$ in comparison with the exact solution (17, 18) for u(x,t) and v(x,t) for the approximate solution of the WBK Eq. 1

x/t	0	0.2	0.4	0.6	0.8	1
$\left|u_{exact}-u_{VIM}\right|=\psi$						
-100	-5.5 E-10	-1.3 E-10	-1.5 E-10	-5.2 E-10	-1.0 E-10	-1.0 E-10
-80	0.0	0.0	-4.2 E-10	0.0	0.0	-1.0 E-10
-60	-2.1 E-10	-1.4 E-10	-1.7 E-10	2.6 E-10	-4.0 E-10	-4.0 E-10
-40	-9.4 E-10	-1.0 E-10	-4.3 E-10	-8.3 E-10	-5.1 E-10	-6.2 E-10
-20	2.0 E-10	-7.0 E-10	-6.0 E-10	6.9 E-10	1.1 E-9	3.3 E-10
0	-5.3 E-10	-1.5 E-10	-1.2 E-10	-7.5 E-10	-1.2 E-9	2.6 E-9
20	5.0 E-10	1.3 E-10	7.0 E-10	5.1 E-10	7.2 E-10	8.2 E-10
40	2.4 E-10	2.1 E-10	2.5 E-10	2.6 E-10	-1.7 E-10	-1.2 E-10
60	5.1 E-1	3.4 E-10	-1.3 E-10	2.5 E-10	2.3 E-10	-2.0 E-10
80	5.6 E-10	5.6 E-10	5 E-10	5.1 E-10	5.9 E-10	5.1 E-10
100	0.0	0.0	0.0	0.0	0.0	0.0
$\left|v_{exact}-v_{VIM}\right|=\varphi$						
-100	3.0 E-18	-1.6 E-12	-3.2 E-14	-4.8E-12	-6.4 E-12	-8.1E-12
-80	-3.0 E-16	-8.8 E-11	-1.7 E-10	-2.6 E-10	-3.5 E-10	-4.4 E-10
-60	1.0 E-14	-4.8 E-9	-9.6 E-9	-1.4 E-8	-1.9 E-8	-2.4 E-8
-40	1.0 E-12	-2.6 E-7	-5.3 E-7	-7.9 E-7	-1.0 E-6	-1.3 E-6
-20	1.0 E-10	-2.5 E-5	-5.0 E-5	-7.5 E-5	-1.0 E-4	-1.2 E-4
0	1.0 E-12	2.5 E-5	5.0 E-5	7.5 E-5	1.0 E-4	1.2 E-4
20	1.0 E-14	2.6 E-7	5.3 E-7	7.9 E-7	1.0 E-6	1.3 E-6
40	-3.0 E-16	4.8 E-9	9.6 E-9	1.4 E-8	1.9 E-8	2.4 E-8
60	3.0 E-16	8.8 E-11	1.7 E-10	2.6 E-10	3.5 E-10	4.4 E-10
80	3.0 E-18	1.6 E-12	3.2 E-12	4.8 E-12	6.4 E-12	8.1 E-12
100	-1.0 E-19	2.9 E-14	5.9 E-14	8.9 E-14	1.1 E-13	1.4 E-13

Table 2: The numerical results for $\left|u_{exact}-u_{VIM}\right|=\psi$ and $\left|v_{exact}-v_{VIM}\right|=\varphi$ in comparison with the exact solution (30, 31) for u(x,t) and v(x,t) for the approximate solution of the MB Eq. 19

x/t	0	0.2	0.4	0.6	0.8	1
$\left|u_{exact}-u_{VIM}\right|=\psi$						
-100	-1.0 E-10	-1.0 E-10	-1.0 E-10	-1.0 E-10	-1.0 E-10	-1.0 E-10
-80	0.0	0.0	3.0 E-10	3.0 E-10	3.0 E-10	5.0 E-10
-60	0.0	3.7 E-10	7.3 E-9	1.0 E-8	1.4 E-8	1.8 E-8
-40	-5.2 E-10	1.9 E-7	3.9 E-7	5.9 E-7	7.9 E-7	9.9 E-7
-20	-4.4 E-10	1.4 E-5	2.8 E-5	4.3 E-5	5.7 E-5	7.2 E-5
0	2.1 E-10	1.4 E-5	2.8 E-5	4.3 E-5	5.7 E-5	7.2 E-5
20	3.8 E-10	1.9 E-7	3.9 E-7	5.9 E-7	7.9 E-7	9.9 E-7
40	0.0	3.7 E-9	7.3 E-9	1.0 E-8	1.4 E-8	1.8 E-8
60	1.0 E-10	1.0 E-10	2.3 E-10	2.9 E-10	3.7 E-10	4.5 E-10
80	1.0 E-10	1.0 E-10	1.0 E-10	1.0 E-10	1.0 E-10	1.0 E-10
100	-1.0 E-10	-1.0 E-10	-1.0 E-10	-1.0 E-10	-1.0 E-10	-1.0 E-10
$\left|v_{exact}-v_{VIM}\right|=\varphi$						
-100	1.0 E-18	-2.4 E-13	-4.8 E-13	-7.3 E-13	-9.7 E-13	-1.2 E-12
-80	-3.1 E-17	-1.3 E-11	-2.6 E-11	-3.9 E-11	-5.3-11	-6.6 E-11
-60	1.0 E-15	-7.20E-10	-1.4 E-9	-2.1 E-9	-2.9 E-9	-3.6 E-9
-40	2.7 E-13	-4.0 E-8	-8.0 E-8	-1.2 E-7	-1.6 E-7	-2.0 E-7
-20	1.6 E-11	-3.8 E-5	-7.6 E-5	-1.1 E-5	-1.5 E-5	-1.8 E-5
0	1.3 E-11	3.8 E-5	7.6 E-5	1.1 E-5	1.5 E-5	1.9 E-5
20	2.2 E-13	4.0 E-8	8.0 E-8	1.2 E-7	1.6 E-7	2.0 E-7
40	1.1 E-15	7.2 E-10	1.4 E-9	2.1 E-9	2.9 E-9	3.6 E-9
60	-3.3 E-17	1.3 E-11	2.6 E-11	3.9 E-11	5.3 E-11	6.6 E-11
80	1.6 E-18	2.4 E-13	4.8 E-13	7.3 E-13	9.7 E-13	1.2 E-12
100	-2.4 E-20	4.4 E-15	8.9 E-15	1.3 E-14	1.7 E-14	2.2 E-14

Table 3: The numerical results for $|u_{exact} - u_{VIM}| = \psi$ and $|v_{exact} - v_{VIM}| = \varphi$ in comparison with the exact solution (43, 44) for u and υ for the approximate solution of the ALW Eq. 32

| x/t	0	0.2	0.4	0.6	0.8	1
$|u_{exact} - u_{VIM}| = \psi$						
-100	0.0	0.0	0.0	0.0	0.0	0.0
-80	0.0	0.0	-1.0 E-10	0.0	0.0	-2.3 E-10
-60	0.0	-3.1 E-10	-1.1 E9	2.5 E-9	-4.7 E-9	-7.3 E-9
-40	-3.2 E-10	-1.6 E-8	-6.6 E-8	-1.4 E-7	-2.6 E-7	-4.1 E-7
-20	0.0	-4.8 E-6	-1.9 E-5	-4.3 E-5	-7.7 E-5	-1.2 E-4
0	-1.0 E-10	-4.8 E-6	1.9 E-6	4.3 E-5	7.7 E-5	1.2 E-4
20	1.9 E-10	1.6 E-8	6.6 E-8	1.4 E-7	2.6 E-7	4.1 E-7
40	0.0	3.4 E-10	1.1 E-9	2.6 E-9	4.6 E-9	7.2 E-9
60	3.0 E-11	7.2 E-11	1.1 E-11	7.6 E-11	1.5 E-10	1.3 E-10
80	1.0 E-10	1.0 E-10	1.0 E-10	1.0 E-10	1.0 E-10	1.1 E-10
100	-5.0 E-11	-5.0 E-11	-5.1 E-11	-5.3 E-11	-5.7 E-11	-5.9 E-11
$|v_{exact} - v_{VIM}| = \varphi$						
-100	9.7 E-12	4.8 E-12	9.7 E-12	1.4 E-11	1.9 E-11	2.4 E-11
-80	5.3 E-10	2.6 E-10	5.3 E-10	7.9 E-10	1.0 E-9	1.3 E-9
-60	2.9 E-8	1.4 E-8	2.9 E-8	4.3 E-8	5.8 E-8	7.2 E-8
-40	1.6 E-6	8.0 E-7	1.6 E-6	2.4 E-6	3.2 E-6	4.0 E-6
-20	2.4 E-4	1.2 E-4	2.4 E-4	3.6 E-4	4.9 E-4	6.1 E-4
0	2.4 E-4	1.2 E-4	2.4 E-4	3.6 E-4	4.9 E-4	6.1 E-4
20	1.6 E-6	8.0 E-7	1.6 E-6	2.4 E-6	3.2 E-6	4.0 E-6
40	2.9 E-8	1.4 E-8	2.9 E-8	4.3 E-8	5.8 E-8	7.2 E-8
60	5.3 E-10	2.6 E-10	5.3 E-10	7.9 E-10	1.0 E-9	1.3 E-9
80	9.7 E-12	4.8 E-12	9.7 E-12	1.4 E-11	1.9 E-11	2.4 E-11
100	1.7 E-13	8.9 E-14	1.7 E-13	2.6 E-13	3.5 E-13	4.4 E-13
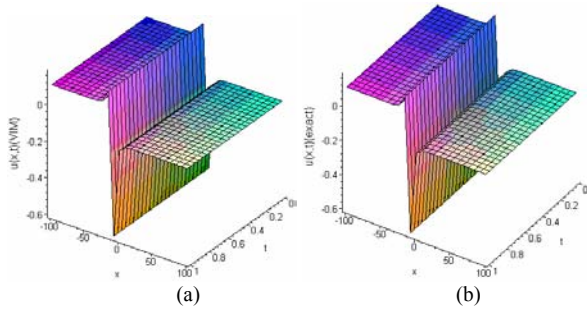
Fig. 5: Propagation of u(x,t) for Eq. 32, VIM (a) and exact solution (b) for k = 0.1, ω = 0.005, α = 0, β = 1.2, x_0 = 10
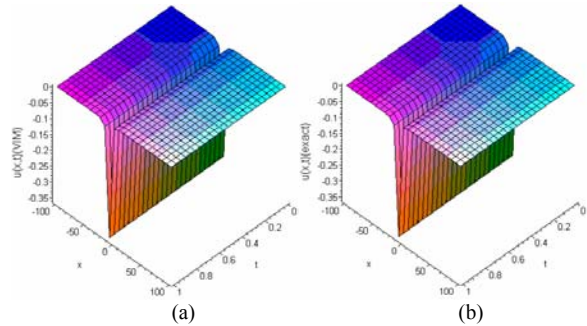

Fig. 6: Propagation of v(x,t) for Eq. 32, VIM (a) and exact solution (b) for k = 0.1, ω = 0.005, α = 0, β = 1.2, x_0 = 10

CONCLUSION

In this research, He's variational iteration method has been successfully implemented to find the analytical solution of nonlinear Whitham-Broer-Kaup (WBK), Modified Boussinesq (MB) and Approximate Long Wave (ALW) equations. All the examples show that the results of the present method are in excellent agreement with those of the exact solutions. Some of the advantages of proposed approach are freely choosing initial conditions with some unknown parameters, so it is capable to achieve the unknown parameters in the initial solution. It also is capable to converge to correct results with fewest number of iterations or even once, for some cases.

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