American Journal of Engineering and Applied Sciences

Original Research Paper

Low Reynolds Number Wing Design for Unmanned Aerial Vehicle: A Case Study

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Article history
Received:01-10-202
Revised: 10-11-22
Accepted: 15-11-2022

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Abstract: With the widespread utilization of Unmanned Aerial Vehicles (UAVs) in many fields, it is essential to identify the parameters governing their design process. By taking the wing as a showcase, this study intends to guide through the design process of the wing, elaborate on some important definitions, and show how different parts of an aircraft affect each other. The current case study is limited to low Reynolds number (200,000: 500,000) wing design for unmanned aerial vehicle. The final wing was designed to be rectangular, a high wing with a span of 2 m, a chord of 0.4 m, and a corresponding aspect ratio of 5 with a total take-off weight of 10 kg. While the cruising speed and stall speeds were 14 and 11 m/s respectively.

Keywords: UAV, Wing Design, Low Reynolds Number

Introduction

The aeronautical engineering major began to grow exponentially in the last 150 years in many different fields. This rapid development and high demand for aircraft with specific missions prompted the scientific community to research and discover the field of aviation more broadly than before in terms of aircraft mission (Gudmundsson, 2021) military (Polhamus, 1984), erobatic, etc., aircraft speed (subsonic (Sadraey, 2012), transonic, supersonic (Kaattari and Wiggins, 1961), etc.), and with technology revolution we had, the unmanned aerial vehicle starts to take place in one of the most desired products of the industrial field (Keane and Carr, 2013; Fahlstrom *et al*., 2022).

One of the most effective parts of the airplane's performance is the wing which has many planform shapes (rectangular (Sadraey, 2012), tapered (Anderson, 1936; 1938), elliptical, triangular (Razak and Snyder, 1966), and even irregular shapes (Ball and Watson, 1976; Spencer, 1961) and gets affected directly by the aircraft mission. For the wings to perform efficiently, aerodynamicists try to estimate the aerodynamic characteristics (lift (Nicolai and Carichner, 2010), drag (Hoerner, 1965), pitching moment, etc.), of those possibilities of wing configurations theoretically or experimentally using aerodynamic tools and wind tunnels. Lift force for example has got many theories of lift for over a hundred years now based on Newton's 3rd law, Bernoulli's principle, Kutta-Zhukovsky condition for sharp edge airfoils (Hoffman and Johnson, 2008; Babinsky, 2003; Coutinho, 2014; McLean, 2012). Moreover, the amazing work of the team at the University of California with a new theory of lift "a variational theory of lift" seems to solve a great part of the puzzle and will create new chapters of aerodynamic books (Gonzalez and Taha, 2022). With all these variables, the design of the UAV's wing can take place within the same Reynolds number range and similar mission of an existing aircraft design method based on aircraft design references. Therefore, the current work aims to show a case of wing design for a CARGO UAV aircraft at a low Reynolds number (200,000-500,000) through the different design phases (conceptual design, preliminary design, detailed design, and performance analysis) with a limited take-off run under 100 ft.

Design Phases

The design process of an aircraft goes through three sequential phases, where every phase's product is the starting point of the next phase. The design process starts with the conceptual design phase where the main goal is selecting an applicable and appropriate concept for the target mission of the aircraft and trying to optimize that concept as much as possible. This phase should be built based on understanding the required mission of the aircraft and using knowledge gained by reading references and design reports or even watching documentaries of aircraft design for similar aircraft missions in addition to previous experience to ensure the generation of the best possible candidates for the conceptual design phase. After that comes the preliminary phase where all constraints can be taken into consideration and the best concept aircraft satisfying the requirements will be involved in some iterations to determine the initial (dimensions, required thrust, material, and estimated weight) of the aircraft. Constraints can be a take-off distance, rate of climb, stall speed, available power, or any other constraint required by the mission or the client. By determining wing loading and wing area from the previous phase, the detailed design can now begin. The detailed design phase is where all the parameters of the wing can be selected including the airfoil by getting into a loop that contains both Sadreay's and Nicolai's methods to convert from the 3D wing to the 2D airfoil characteristics and back. Performance analysis will come after as a final part of the detailed design phase to make sure that the aircraft is efficient to perform the required mission. In the current case of a CARGO aircraft, a Technical Data Sheet (TDS) can be generated of the payload prediction curve to predict the ability to lift the payload of the aircraft with the change of the density altitude. Then the configuration modification is frozen, and the decision has been made to build the aircraft. At this point in the design process, everything is ready to take off. CAD drawings and manufacturing methods all of these are ready to start the manufacturing process to produce the state-of-the-art which is used as a prototype to refine a later production (Nicolai and Carichner, 2010). That is a brief about the design phases which will be further elaborated on in the next sections. Figure 1 shows an overview of the aircraft wing design procedures.

Conceptual Design

The design process of an aircraft includes four main phases: (i) Conceptual design, (ii) preliminary design, (iii) detailed design, and (iv) evaluation. The conceptual design phase comes first with the highest priority in the aircraft system design and development process. It is an early lifecycle activity to achieve and otherwise predetermine the function, shape, cost, and development sequence of the required aircraft system. The associated definition of need is the best starting point for the conceptual design phase. The main responsibility of the conceptual design is to be responsive to the customer's identified requirements by selecting the path forward for the design and development of a preferred system configuration Fig. 2.

Aircraft Wing Configuration Alternatives

Once the necessary aircraft components have been determined and a list of the major components is prepared, then the configurations of the components are determined. Several alternatives for each component carry advantages and disadvantages by which the design requirements are satisfied at different levels. Since each design requirement has a unique weight, each configuration alternative results in a different level of satisfaction (Gudmundsson, 2013). The wing configuration alternatives from three different aspects are as follows: Number of wings, wing location, and wing type Fig. 3.

Table 1: Wing configuration selection

Criteria	Weighting	Rectangular	Tapered	Midway Taper	Elliptical
Lift coefficient	0.35	0.8000	0.850	0.800	0.9000
Manufacturing	0.30	0.9000	0.700	0.750	0.5500
Induced drag	0.20	0.7000	0.800	0.750	0.9000
Wing weight	0.15	0.7500	0.850	0.800	0.8500
Total	1.00	0.8025	0.795	0.775	0.7875

Table 2: Wing vertical location selection

Criteria	Weighting	High wing	Mid wing	Low wing
Stability	0.3	0.90	0.80	0.60
Ground effect	0.3	0.60	0.80	0.90
Ground clearance	0.2	0.90	0.70	0.60
Structural weight	0.1	0.80	0.60	0.90
Downwash on tail	0.1	0.60	0.80	0.90
Total	1.0	0.77	0.76	0.75

Table 3: The performance parameters for the current design

Performance parameter	Value
Aircraft cruise speed	14.0 m/s
Stall speed	11.0 m/s
Rate of climb	1.5 m/s
Take-off distance	30.0 m
Constant velocity turn	45°

Table 4: Wing airfoil trade-off

Criteria	Weighting	S1223	LNV109A	FX 74-CL6-140	E423
Clmax	0.35	0.78	0.67	0.73	0.75
L/D @ Cl_i	0.30	0.65	0.75	0.85	0.90
Stall behavior	0.20	0.70	0.70	0.70	0.75
Cm	0.15	0.60	0.90	0.75	0.65
Total	1.00	0.70	0.74	0.76	0.78

Number of Wings

An aircraft is also categorized based on the number of wings, it could have up to three wings, the monoplane is almost the most common and practical option in conventional modern aircraft which will also be adopted in this study. The biplane and triplane were the most popular configuration in WWI and WWII due to the limitation in manufacturing technology and lack of knowledge with the primary advantage of having a large wing area that can be packed in a small wingspan (with the same total area) leading to highly maneuverable airplanes with relatively low stalling speeds without flaps. Old manufacturing technologies were not able to structurally support a long wing, to stay level and rigid. With the advancement in manufacturing technologies and the development of new strong aerospace materials and airfoils, the use of triplane and biplane configurations was mostly dropped (Gudmundsson, 2013).

Wing Type

The wing shape is crucial in the aircraft's aerodynamic performance, weight, and manufacturability. Although the elliptical wing has the ideal lift distribution, its fabrication was the barrier. Therefore, every criterion should be weighted based on knowledge and manufacturing capabilities to guarantee an optimum selection, as shown in Table 1.

Wing Mounting Position

Despite the low difference between the high and medium wing options, the medium wing is characterized by a better take-off performance, better horizon view, less induced drag, and higher lateral control, on the other hand, the high wing is favorable to increase stability, safety during take-off and landing, and a safer configuration in terms of ground clearance to avoid structural damage while take-off and landing Table 2.

Preliminary Design

Initiating the design process of an aircraft is a real challenge, where a huge number of parameters should be assumed in reasonable ranges to be able to reduce the iterative design time. So, one of the first tasks performed in the design process is constraint analysis using a special diagram called "Constraint analysis diagram", its primary advantage is to determine the required wing area and power plant for the design to meet all the performance requirements Gudmundsson, 2013). The two axes of the diagram are Thrust-to-Weight ratio (T/W) on the y-axis and wing loading (W/S) on the x-axis. Any combination that is above the constraint curves will satisfy the performance requirements. The main idea is to draw iso curves that have the same value of a certain parameter but in the shape of a function of T/W and W/S Gudmundsson, 2013). Some of these constraints can be plotted by the following equations:

$\frac{T}{W}$ for a level constant-velocity turn:

$$\frac{T}{W}=q\left[\frac{CD_{\min}}{(w/s)}+\left(\frac{n}{q}\right)^2\left(\frac{W}{S}\right)\right]$$

$\frac{T}{W}$ for a desired specific energy level:

$$\frac{T}{W}=q\left[\frac{CD_{\min}}{(w/s)}+\left(\frac{n}{q}\right)^2\left(\frac{W}{S}\right)\right]+\frac{P_S}{V}$$

$\frac{T}{W}$ for a desired rate of climb:

$$\frac{T}{W}=\frac{V_V}{V}+\frac{q}{(W/S)}CD_{\min}+\frac{k}{q}\left(\frac{W}{S}\right)$$

$\frac{T}{W}$ for the desired Take-Off distance:

$$\frac{T}{W}=\frac{V^2{}_{LOF}}{2g.S_G}+\frac{q.CD_{TO}}{(W/S)}+\mu\left(1-\frac{q.CL_{TO}}{(W/S)}\right)$$

$\frac{T}{W}$ for the desired cruise airspeed:

$$\frac{T}{W}=qCD_{\min}\left(\frac{1}{(W/S)\_}\right)+\left(\frac{1}{q}\right)\left(\frac{W}{S}\right)$$

$\frac{T}{W}$ for a Service Ceiling (Rate of Climb = 1.667 ft/s or 0.508 m/s):

$$\frac{V_v}{\sqrt{\frac{2}{\rho}\frac{w}{S}\sqrt{\frac{k}{3CD_{\min}}}}}+4\sqrt{\frac{k.CD_{\min}}{3}}$$

For the current case study, the performance parameters were chosen as tabulated in Table 3.

Using the values above resulted in a T/W ratio = 0.31 and W/S = 15.1355, as shown in the Constraint diagram Fig. 4.

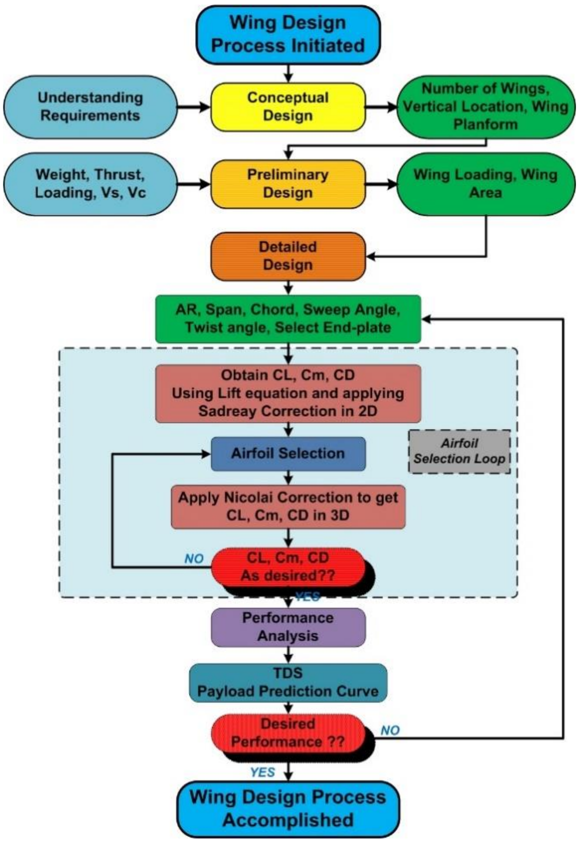


Fig. 1: Overview of the Aircraft wing design procedures

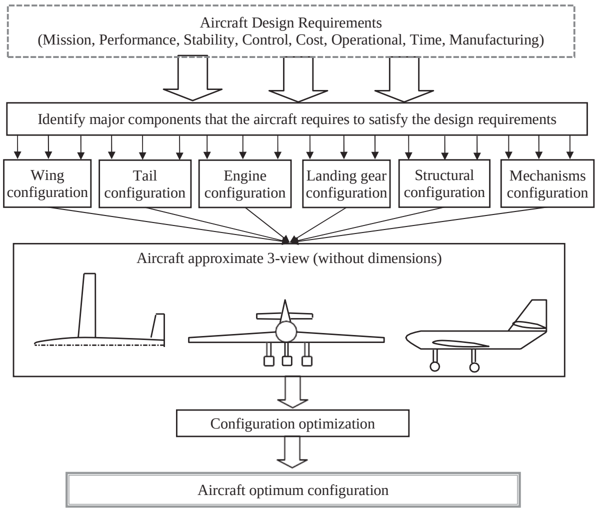


Fig. 2: Illustration of the major activities which are practiced in the conceptual design phase (Sadraey, 2012)

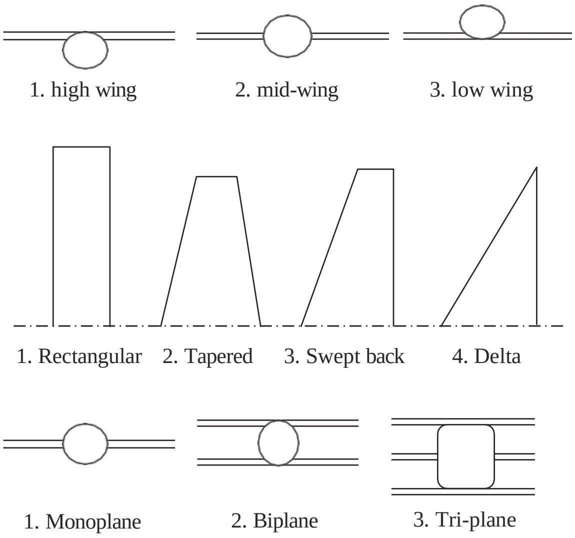


Fig. 3: Illustration of several wing configuration alternatives (Sadraey, 2012)

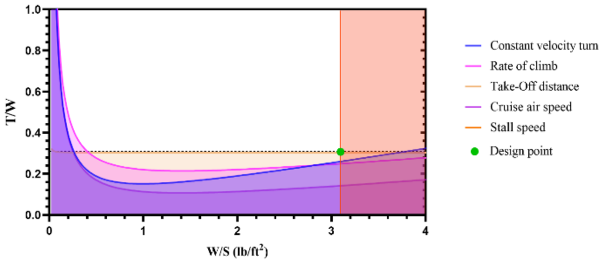


Fig. 4: Constraint diagram that shows T/W versus wing loading W/S

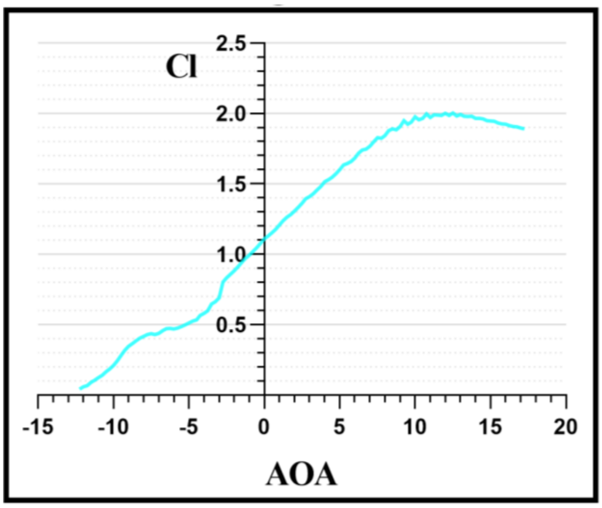


Fig. 5: E423 airfoil Cl vs Alpha @ 200,000 Reynolds number (http://airfoiltools.com/)

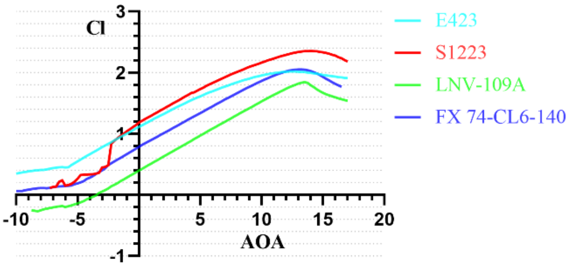


Fig. 6: Analytical airfoils data by XFLR5

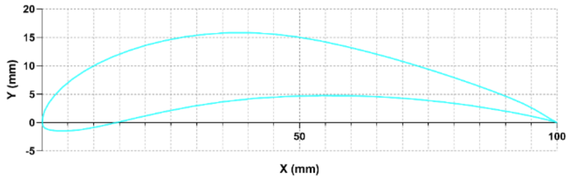


Fig. 7: E423 airfoil geometry (Anderson, 1936)

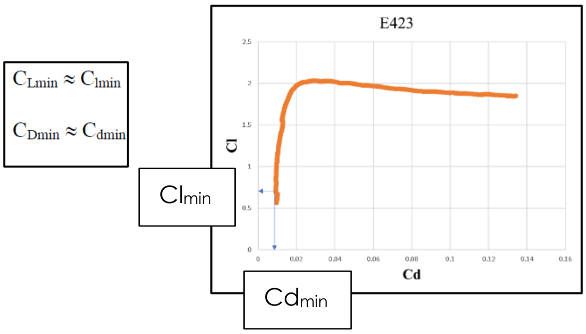
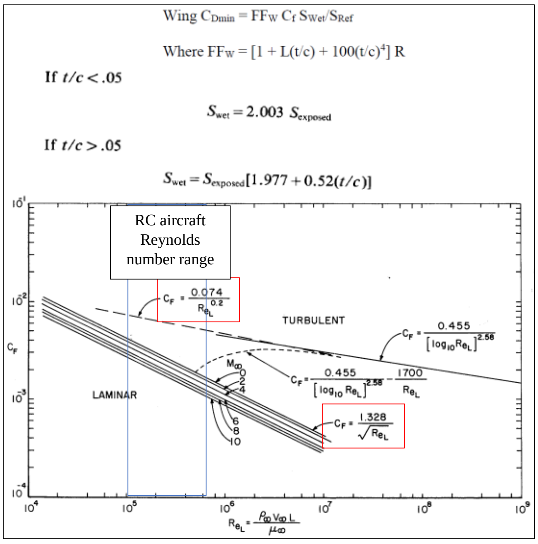


Fig. 8: Graphical method to estimate the pressure and skin friction drag coefficient



Wing $C_{Dmin} = FF_W C_f S_{Wet}/S_{Ref}$

Where $FF_W = [1 + L(t/c) + 100(t/c)^4]\, R$

If $t/c < .05$

$$S_{wet} = 2.003\, S_{exposed}$$

If $t/c > .05$

$$S_{wet} = S_{exposed}[1.977 + 0.52(t/c)]$$

Fig. 9: Skin friction Coefficient (Cf) versus Reynolds Number (Vural and Nicolai, 2009)

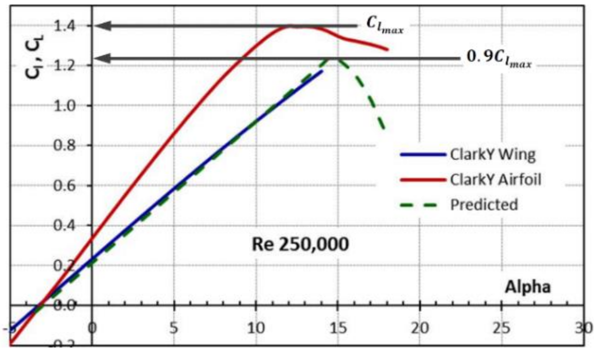


Fig. 10: Predicted wing CL for ClarkY airfoil by Nicolai (Vural and Nicolai, 2009)

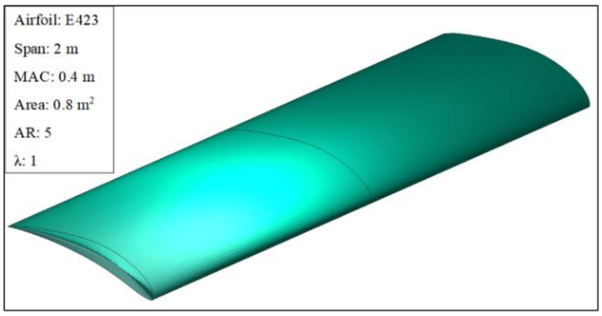


Fig. 11: Final designed wing configuration

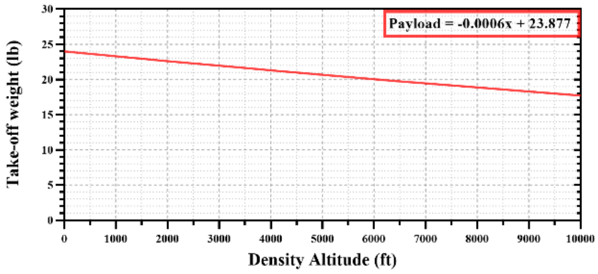


Fig. 12: Total Take-off weight prediction curve

Detailed Design Phase

For the detailed design phase, the area, and velocity (minimum area S = 0.6607 m^2, $V_S = 11$ m/s, $V_{TOF} = 13.2$ m/s $V_C = 14$ m/s) of (stall, take-off, and cruise) conditions were detected by constraint diagram in preliminary phase then the detailed design phase could begin with:

Aspect Ratio (AR) Determination

It is mandatory to know that the Aspect Ratio (AR) is a main effecting parameter on the aircraft performance especially when it comes to power plant performance calculations. As AR increases the lift increases while induced drag decreases. For the current case, a wing with an AR of 5 is selected (Raymer, 2012).

Detecting the Required Wing Lift Coefficient (CL) for Various Conditions

Detect the required lift Coefficient (CL) of (stall, take-off, and cruise) to get the airfoil coefficient of lift (cl) required for all conditions to examine for the suitable airfoil in the next step (according to the following equation):

$$m.g = 1/2 \times \rho \times V^2 \times S \times CL$$

- Note: CL is a 3D wing Characteristic, not a 2D cl that refers to airfoils
- For each flight condition, there is a value for aircraft velocity and a corresponding CL required for that flight condition
- $V_{\text{Take-Off}} = k \times V_{\text{Stall}}$, where k is about 1.1 for fighter aircraft and about 1.2 for jet transports and GA aircraft (Sadraey, 2012)
- Velocity values were given by the propulsion sub-team and might be determined by a customer’s desire or capabilities for an existing propulsion system
- For the current case study: The aircraft's total take-off weight was 10 kg, the wing area was 0.8 m^2, at sea level so $\rho = 1.225$ Kg/m^3, and the velocity value for the stall, take-off, and cruise were (11, 13.2, and 14 m/s) respectively, then the minimum CL required for the stall, take-off and cruise were (1.66, 1.149 and 1.021) respectively

To Get the Required (cl) from the 3D CL

Sadraey correction was used:

- cl = CL/(0.9 * 0.95) (Sadraey, 2012)
- Stall: $cl_{max} = 1.9352$
- Take-off: $cl_{TOF} = 1.343$
- Cruise: $cl_i = 1.1947$

Airfoil Selection

The subsequent step was to select an airfoil that fulfills the minimum requirements. This was realized with the aid of the XFLR5 software and a trade-off matrix to select the optimum airfoil (http://airfoiltools.com/).

For heavy cargo aircraft, it's desired to select a high-lift airfoil to minimize the wing area. For the current case study, an E423 airfoil which is a high-lift airfoil with capabilities to operate in low Reynolds numbers was implemented. E423 is the kind of airfoil that can't be flapped due to high camper geometry that suffers laminar stall at low Reynolds number (Fig. 5, 6 and 7) (Eppler, 1990; Selig and Guglielmo, 1997). However flapped airfoil can be applied, and its effect can be calculated by flap span to wingspan as a percentage multiplied by the delta of lift coefficient of the fully flapped wing and none flapped wing (XFLR5 can be used for this purpose) and this effect can be added to the actual none flapped wing to get lift coefficient at flapped wing Table 4.

Calculate the 3D Characteristics from the 2D

Now it’s time to generate the wing characteristics as follows:

Select an Aspect ratio (5 or higher for better aerodynamic characteristics) and determine span and chord by Sadraey (2012); Raymer (2012):

$$AR = \frac{b}{c}$$

The wing planform area with a rectangular or straight tapered shape is defined as the span times the MAC.

$$S = b \times c$$

Thus, the Aspect Ratio (AR) shall be redefined as:

$$AR = \frac{b \times b}{c \times b} = \frac{b^2}{S}$$

- Based on (Sadraey, 2012; Vural and Nicolai, 2009) $Cl\alpha$ can be 2π as an approximation for all airfoils or calculated by:

$$Cl_\alpha = \frac{dcl}{da} = 1.8\pi\left(1 + 0.8\frac{t}{c}\right)$$

- 3D CL_α can be calculated by the formula (Sadraey, 2012):

$$CL_\alpha = \frac{dcL}{d\alpha} = \frac{Cl_\alpha}{1 + \frac{Cl_\alpha}{\pi + AR}}$$

- Select a suitable incidence angle that fulfills *cli* requirements
- Calculate the Oswald span efficiency, for a rectangular wing (Sadraey, 2012):

$$e = 1.78\left(1 - 0.045AR^{0.68}\right) - 0.64$$

- Calculate the pitching moment (Sadraey, 2012):

$$CM_{O-wf} = CM_{af}\frac{AR\cos^2(\Lambda)}{AR + 2\cos(\Lambda)} + 0.01\alpha_t$$

- where, (α_t) is the twist angle and (Λ) is the wing sweep angle, AR is the wing aspect ratio
- Total drag (Pressure and skin friction drag+Induced drag+Viscous drag due to lift) (Vural and Nicolai, 2009):

$$CD = C_{D\min} + K\,C_L^2 + K''\left(C_L - C_{L\min}\right)^2$$

- Calculate Pressure and skin friction drag (Vural and Nicolai, 2009)
- *Graph method* Fig. 8
- Calculation Method

F_w is the wing form factor, {L} is the airfoil thickness location parameter (L = 1.2 for the max t/c located at ≥0.3 chords and L = 2.0 for the max t/c <0.3 chords), {t/c} maximum thickness ratio and {R} is the lifting surface correlation parameter. For a low speed, unswept wing R is approximately 1.05 Fig. 9:

$$S^{ref} = S_{exposed} = b \times MAC = 0.8m^2$$

$$S_{wet} = 0.8(1,977 + 0.52 \times 0.125) = 1.6336m^2$$

$$FF_w = \left[1 + (2\times 0.125) + 1.05(100 \times 0.125^4)\right] = 1.33812$$

Assume laminar:

$$C_F = \frac{1.328}{\sqrt{350440}} = 2.2433\times 10^{-3} \ \&\ CD_{\min} = 0.006129$$

Note that:

- In turbulent case, C_f can be calculated by one of two equations based on the Reynolds number
- Reynolds number can differ as velocity changes based on aircraft condition so parasite drag can be calculated in each condition
- C_f can be calculated by assuming flow laminar or turbulent
- Calculate the induced drag for each condition (Sadraey, 2012):

$$K = \frac{1}{\pi . e . AR} \ \&\ CD = K.CL^2$$

- Viscous drag-due-to-lift:
  Based on Nicolai's paper this kind of drag was hard to estimate, and had very small drag compared to other types of drag thus, it was often omitted
- Calculate CL for wing by Leland Nicolai's recommendation
  From the angle of zero lift (α_{C0L}), a line was drawn with a slope of CLα till the line intersects with the value of 0.9* $C_{l\max}$ of the airfoil. This corresponding line is the predicted 3D lift for our wing Fig. 10

Final Wing Dimensions

The final dimensions and parameters of the wing are summarized in Table 5. Figure 11 illustrates the final wing configuration with data.

Performance Study

It is significant to detect the performance of the aircraft as the density altitude changes with air density, air temperature, air pressure, and humidity and thus, affecting sufficiently the capability of the aircraft to carry payloads. More details about density altitude and how it can be affected by weather conditions can be gained by reading Pilot's Handbook of Aeronautical Knowledge (Duncan, 2016).

Table 5: Final wing dimensions and parameters

Wing planform			3D parameters		
Wing Area	0.8 m^2	α incidence angle	0 degrees	CL TOF	1.149000
Span	2.0 m	Clα (2D)	6.22040	CDo @TOF	0.006316
MAC	0.4 m	CLα (1/rad)	4.45580	CDi @ TOF	0.093300
Taper Ratio	1.0	Cm_ac	-0.17140	CD_{TOF}	0.099616
Vc	14 m/s	CL cruise	1.021400	CL_{max}	1.800000
Vtof	13.2 m/s	CDo	0.006129	CD_{max}	0.236250
Vs	11 m/s	CDi @ cruise	0.073700	e	0.900700
RE @ Cruise & Stall	350440 &250000	CD cruise	0.079829	Airfoil	E423

Total Take-off Weight vs Density Altitude

Some are preferring to predict payload weight against density altitude while others are preferring to calculate total take-off weight against density altitude, however, we prefer to calculate the total take-off weight due to the uncertainty in the calculation if the prediction is based on payload weight due to maintenance operations which can sufficiently be affecting aircraft empty weight thus, gives a false reading of the allowable payload weight.

Technical Data Sheet (TDS) Generating Steps

The take-off weight against density altitude could be predicted as follows:

- Get your elevation above sea level (for example Alexandria, Egypt 16.4 ft above sea level)
- Get weather conditions for your actual location by any measuring device or through online websites (for example 5 AM 21 May 2022 Alexandria, Egypt) (https://www.wunderground.com/history/daily/HEAX/date/2022-5-21)
- Use online calculators (https://wahiduddin.net/calc/calc_da.htm.) to calculate density and density altitude based on weather conditions. Apply these data on the lift equation (https://www.grc.nasa.gov/www/k-12/airplane/lifteq.html) (for most critical conditions, usually stall condition) to calculate total take-off weight
- Finally, a total take-off weight prediction curve can be plotted and ready to be used in the event of flying the aircraft Fig. 12

One can see the capability of the aircraft is way higher (10.8789 kg) than the design condition (10 kg) due to the difference between the minimum CL_{max} allowable (1.6546) and the actual CL_{max} of the E423 (1.8).

Conclusion

Unmanned aerial vehicles are today one of the most required technologies in the 21st century in different fields such as package delivery, navigation, path planning, and firefighting. As the UAV mission differs, the whole aircraft design is affected including the wing design. This research work intends to show a case of wing design for a CARGO UAV aircraft at a low Reynolds number (200,000-500,000) through the different design phases (conceptual design, preliminary design, detailed design, and performance analysis) with a limited take-off run under 100 ft. The final aircraft's wing was designed with a total take-off weight of 10 kg, while the wingspan, AR, Reynolds number @ cruise, and stall speed are 2 m, 5, 350440 & 250000 respectively.

Acknowledgment

The entire authors are utterly grateful to Alexandria University for providing the technical and financial support for this study.

Funding Information

The authors have not received any financial support or funding to report.

Author’s Contributions

Mohammed El-Adawy: Designed the research plan and organized the study and supervision.

Alhassan H. Farid, Mohamed Ahmed Hassan, Mahmoud Abady, Donia Medhat, Habiba Abdullatif, Omar Hassan and Salaheldin Mohamed Thabet: Participated in all experiments, coordinated the data-analysis and contributed to the writing of the manuscript.

Ethics

The corresponding author confirms that all other authors have read and approved the manuscript and no ethical issues have been involved.

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List of Symbols:

AR:	Wing Aspect ratio
b:	Span
C:	Mean aerodynamic chord
CD:	Wing Total drag (Induced Pressure and skin friction drag)
CDi:	Wing Induced drag
CD_{min}:	Wing Pressure and skin friction drag
CD_{TOF}:	Drag coefficient during the Take-Off run
C_{fe}:	Skin friction coefficient
CL:	Wing coefficient of lift
cl:	Airfoil coefficient of lift
CL_{TOF}:	Lift coefficient during the Take-Off run
CL_{max}:	Wing lift coefficient in a stall condition
C_{Lmin}:	CL for minimum wing drag
CL_{α}:	3D wing lift curve slope
Cl_{α}:	2D airfoil lift curve slope
Cm_{af}:	Airfoil section pitching moment coefficient
Cm_{ac}:	Coefficient of bitching moment around aerodynamic center
$Cm_{o\text{-}wf}$:	Wing/fuselage pitching moment coefficient
D:	Wing drag force
e:	Oswald efficiency factor
FF_{w}:	Wing form factor
g:	Standard acceleration of gravity
K:	Lift-Induced drag constant

K'':	Viscous factor
L:	Airfoil thickness location parameter
M:	Wing pitching moment
m:	Mass = W/g
n:	load factor = 1/cos Φ
Ps:	specific energy level at the condition
q:	dynamic pressure at the selected airspeed and altitude (lbf/ft^2 or N/m^2)
R:	Lifting surface correlation parameter
Re:	Reynolds number
S:	Wing reference area = exposed area = b * C
S_G:	Ground run
S_{ref}:	Wing reference area
S_{wet}:	Wing-wetted area
T:	Thrust (Ibf or N)
t/c:	Maximum thickness-to-chord ratio of the airfoil
T/W:	Thrust-to-weight ratio
V:	airspeed
V_C:	Aircraft cruise speed
V_{LOF}:	Liftoff speed
V_S:	Aircraft stall speed
V_{TOF}:	Aircraft take-off speed
V_V:	Vertical speed or (Rate of Climb) = 1.667 ft/s
W:	weight (lbf or N)
W/S:	wing loading
ρ:	Density (mass per unit volume)
Λ:	Sweep angle
μ:	Ground friction constant
α_{C0L}:	Angle of zero lift
α_t:	Twist angle