American Journal of Engineering and Applied Sciences

Original Research Paper

Geometric Optimization of Complex Heat Exchangers using Constructal Technique

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Article history
Received: 28-09-2021
Revised: 08-10-2021
Accepted: 11-10-2021

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Abstract: The application of microchannel heat exchangers for cooling is documented in this study. Numerical optimization is carried out with the goal of minimising the peak wall temperature in the microchannel heat sink modelled with solid and half hollow square micro fins. A high-density heat flux *q''* from a microelectronic device was applied at the bottom wall of the microchannel heat sink with circular flow channel embedded in a highly conductive rectangular block aluminium substrate. Single phase water was used as coolant to remove the deposited heat at the internal channel walls and in the internal surfaces of half hollow square fins, while cool air stream flows externally over the vertical fins in a forced convection laminar regime to take away the excess heat on the micro square fins. Computational Fluid Dynamics (CFD) code was employed to descretised and solve the governing equations. The results of numerical optimization showed that the combined microchannels with solid square fins has the highest global thermal conductance than the microchannel with half hollow square fins. The limiting case of a single microchannel heat sink without fins agrees with what is in open literature.

Keywords: Microchannel Heat Sink, Complex Heat Exchangers, Forced Convection, Numerical Optimization, Conjugate Heat Transfer

Introduction

The idea of cooling microelectronics subject to high heat fluxes and thermal dissipation using microchannel heat sinks, was first carried out by (Tuckerman and Pease, 1981). The novel work led to several research works to improve on thermal performance of microchannel. To further develop and expand on the earliest work done, Chen *et al*. (2009) developed and analysed numerically a three-dimensional model of heat transfer and fluid flow in a noncircular microchannel heat sink. In addition, they performed a comparative analysis of thermal efficiencies in triangular, rectangular and trapezoidal microchannels heat sinks. A numerical optimisation of a silicon microchannel heat sinks with the objective of minimising the thermal resistance and pumping power has been reported (Qu and Mudawar, 2002). Furthermore, more research along the line on microchannel optimisation was carried out by (Husain and Kim, 2008; Salimpour *et al*., 2011; Li *et al*., 2014; Salimpour *et al*., 2013; Bejan and Lorente, 2011; Bejan and Lorente, 2004; Bejan and Lorente, 2006; Olakoyejo *et al*., 2012; Almogbel and Bejan, 2000). The researchers numerically optimised and investigated heat transfer mechanism in microchannel heat sink and searched for the microchannel width that minimises thermal resistance. Geometric optimisation was performed for a microchannel heat sink by fixing the total elemental volume and axial length of the micro-channel heat sink, before carrying out optimisation to obtain best channel dimensions and system configurations (Kou *et al*., 2008). It has been observed that geometric optimisation technique involving constraint specification in studying forced convection heat transfer in microchannels and heat sinks is becoming a widespread approach in understanding thermal management. The idea of geometric optimisation is based on unknown or missing geometry and it begins with global objective (s) and constraint (s) of the flow system. The Constructal law first promulgated by Adrian Bejan is instrumental to the success of geometric optimisation approach and it states that, “For a finite-sise flow system to persist in time, its configuration must change in time such that it provides easier access to its currents (Bejan and Lorente, 2006; Almogbel and Bejan, 2000). Constructal design approach was used to numerically carry out geometric optimisation of 3-D microchannels with three different cross-sections (rectangular, elliptic and isosceles triangle) (Li *et al*., 2014; Lin and Lee, 1997; Bello-Ochende *et al*., 2010). The application of extended surfaces to augment fluid flow and heat transfer is on the increase. Micro pin fins are employed to enhance performance of microchannel heat sinks and this has led to drastic improvement in microchannels thermal efficiency. This study bears some similarities with the investigation by Adewumi *et al*. (2013), who inserted a micro-pin-fins into a rectangular block microchannel channel. But the novel work currently investigated in this study deployed constructal design and theory into fabricating hollows into the micro fins and two cooling media were used. The author claimed that this kind of novel approach using constructal design and theory in microchannels have not been reported in literature. The scheme seeks to obtain suitable optimal geometric configurations that best maximises global thermal conductance the cooling process used two coolants: Single-phase water and air in a forced convection laminar flow. The water and air flows unidirectional and the heat transfer was a conjugate approach, conduction heat transfer in the solid and convection in the fluid.

Geometry and Description of Physical Model

The physical geometry of the microchannel heat sinks of axial length N, height H and width M, was constructed using ANSYS 18.1 (2018) workbench design modeller. The microchannel heat sink was fabricated from a highly conductive aluminium material substrate of thermal conductivity 202.4 W/m K. The first case (Fig. 1a, b and c) has solid square micro fins of equal diameter (fins sides) a_1 and height h_1, placed on the single microchannel heat sink to form a combined microchannel as shown in Fig. 1a, b and c. The characteristic thicknesses and internal structures are t_1, t_2 and t_3. A high-density heat flux q'' was applied at the bottom surface of the combined microchannel and the inlet and outlet temperatures were T_{in} and T_{out}. The computational domain has entrance length, upper free stream region and inlet and outlet planes. The surfaces are far enough from the boundaries and are such that the results are independent of the domain (Rouvreau *et al*., 2005; Velayati and Yaghoubi, 2005). Once, the computational domain is set up, water flows through the microchannel and the cool air stream flows over the square fins in a forced convection laminar condition. Similarly, the second case (Fig. 2a, b and c) has half hollow square micro pin fins mounted on a rectangular block microchannel. The external sides (diameter) are a_1 and the internal sides are a_2, while the external and internal heights are h_1 and h_2 the external diameter a_1 was bored 20 μm to form internal hydraulic diameter a_2 and the internal half hollow fin height was drilled 50% (35 μM) of h_1. The unit cell combined microchannel was inserted into the computational domain (Fig. 1c and 2c) and numerical simulation performed.

Manufacturing Constraints and Design Variables

The global fixed volume of elemental microchannel heat sink is:

$$V_{mch} = WLH \tag{1}$$

The global fixed volume of combined microchannel with fins is:

$$V_{cm} = V_{mch} + V_{f1} + \dots V_{fn} \tag{2}$$

where, V_{fn} is the volume of the fins and n = 1, 2, 3, number of fins attached on top of the microchannel heat sink.

The area of the microchannel heat sink is:

$$A = \frac{V}{N} = WH \tag{3}$$

The aspect ratio is:

$$AR = \frac{H}{W} \tag{4}$$

The solid volume fraction or porosity is:

$$\varnothing = \frac{V_s}{V} \tag{5}$$

The manufacturing constraint of aspect ratio of microchannel with solid square fins is:

$$0.5 \le \frac{h_1}{a_1} \le 4 \tag{6}$$

Governing Equations, Mathematical Modelling and Boundary Conditions

The continuity, momentum and energy equations for cooling fluids are given as follows.

Conservation of mass (continuity):

$$\nabla(\rho\vec{V}) = 0 \tag{7}$$

Conservation of momentum:

$$\vec{V}.\nabla\left(\rho\vec{V}\right) = \nabla\rho + \nabla.\left(\mu\nabla\vec{V}\right) \tag{8}$$

Conservation of energy for fluid:

$$\vec{V}.\nabla(\left(\rho c_p{}^T{}_g\right) = \nabla.\left(k_g\nabla T_g\right) \tag{9}$$

Conservation of energy for the solid:

$$\nabla\left(k_w\nabla T_w\right) = 0 \tag{10}$$

The continuity of the heat flux at the interface between the solid and the liquid is given as:

$$k\,s\frac{\vartheta\tau}{\vartheta n}_{wall}\Bigg| = kf\,\frac{\vartheta\tau}{\vartheta n}\Bigg|_{wall} \quad (11)$$

The no-slip boundary condition is specified for the fluid at the walls of the channel.

Zero stress at the outlet such that:

$$\frac{\vartheta u}{\vartheta x}=0, \frac{\vartheta v}{\vartheta x}=0\,\frac{\vartheta w}{\vartheta x}= \quad (12)$$

The velocity of fluid (water) at the inlet is $u_x = u_{in}$, $v_y = 0$, $w_z = 0$ and the temperature at the entrance and outlet for liquid and air are $T = T_{in}$ and $T = T_{w,L}$.

At the bottom of the microchannel heat sink, the thermal conditions applied are assumed to be:

$$k_s\,\frac{\vartheta T}{\vartheta y} = -q, \quad (13)$$

The measure of performance is given by thermal conductance as:

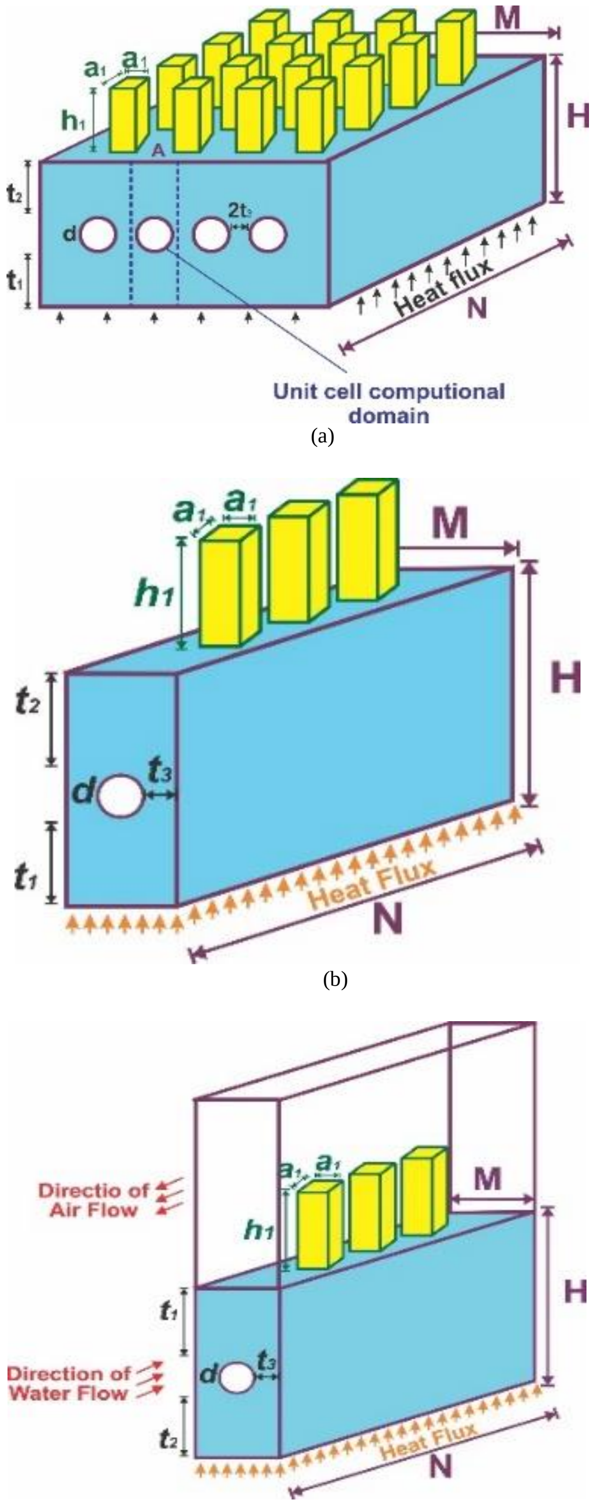
$$C = \frac{q"L}{K_f\left(T_W - T_{in}\right)} \quad (14)$$

Numerical Method, Grid Refinement Test and Code Validation

The continuity, momentum and energy Eq. (7)-(10) were discretised and solved numerically using a 3-D CFD package Lin and Lee (1997), which uses the finite volume method ANSYS FLUENT (2018). The finite volume method divides the computational domain (Fig. 3a and b) into several control volumes as explained by (Pantakar, 1980). The pressure-velocity coupling was executed by SIMPLE algorithm and the second order upwind technique was used to compute the momentum and energy equations.

Grid Refinement Test

Table1 and 2 show the dimensions of the microchannel used for comparison and in this study. The grid refinement test was conducted on each independent case of microchannel with solid and half hollow square fins, to ascertain which mesh sizing and grid size gives accurate results and has less computational time. Hexahedral mesh was used in the discretisation of the domain. The grid refinement test was carried out with the uniform heat flux used in the simulation. The mesh elements were doubled severally until stable mesh elements of 41689 and 54196 nodes, corresponding to the stable maximum outlet wall temperature were obtained. There was less than 1% change in the number of elements within the field for the maximum wall temperature monitored. Any further refinement of the grid shows no remarkable difference in the peak wall temperature.



(a)

(b)

(c)

Fig. 1: Schematic diagram of (a) microchannel heat sink (b) unit cell (c) computational domain

DOI: 10.3844/ajeassp.2022.43.50

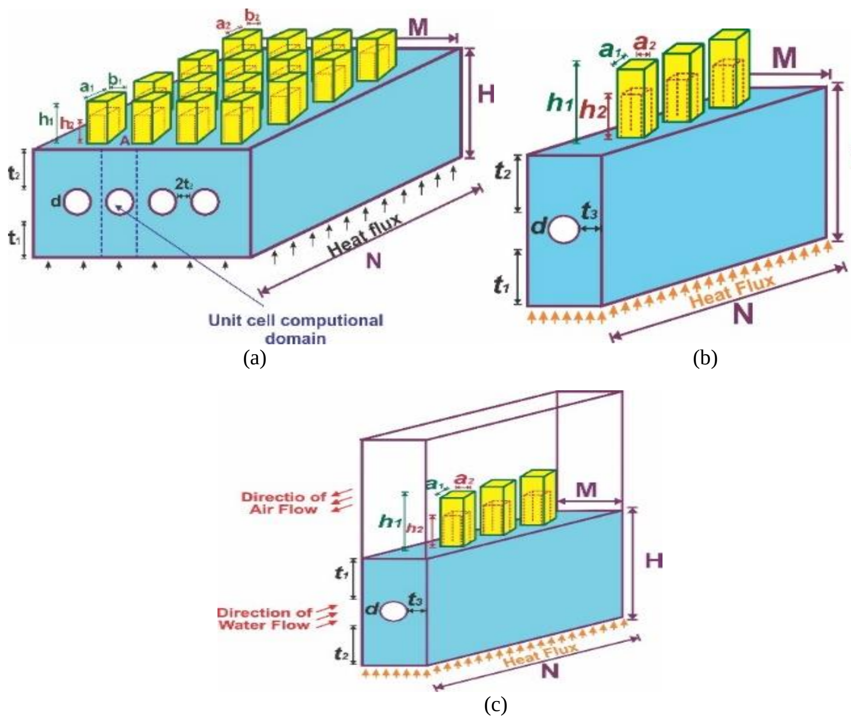


Fig. 2: Schematic diagram of (a) microchannel heat sink (b) unit cell (c) computational domain

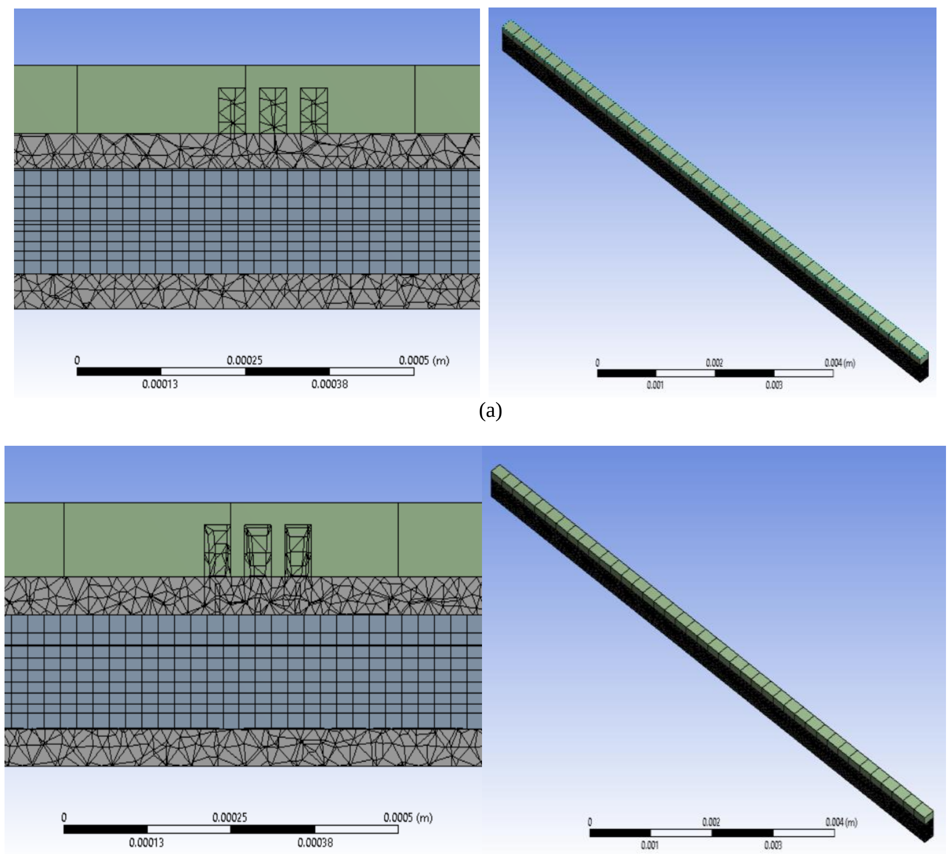


Fig. 3: Discretised domain of microchannel with (a) solid square fins (b) half hollow square fins

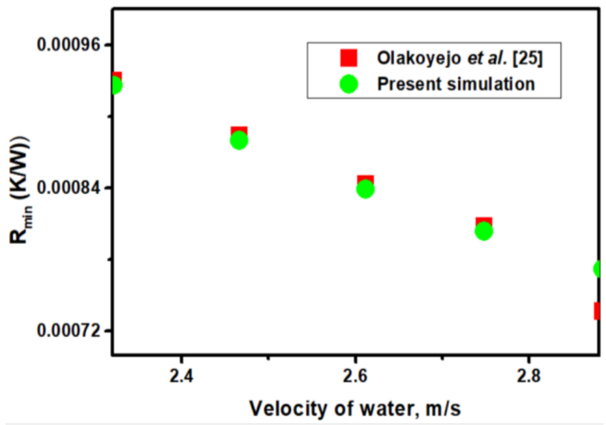


Fig. 4: Numerical code validation: Comparison of present prediction with

Code Validation

The validation of the CFD code used in this numerical optimization is one by comparing the numerical results with results obtained by Olakoyejo *et al*. (2010) and the trends are in good agreement with a deviation of less than 0.001% as shown in Fig. 4. This confirms the validity of the numerical code used and gives confidence in the numerical code.

Optimisation Technique and Methodology

The Design Exploration Optimization Tool in ANSYS (2017) was used to determine the key parameters influencing the design and find the best conditions that minimises peak temperature. The optimization was carried on the microchannel with a fixed axial length of 10 mm and a varying elemental volume V from of 0.38 to 0.5 mm^3. The design space for the response surface was defined as $90 \leq d \leq 154.9$ μm, t_1 and t_2 were 50 μm, while t_2 varies from 46 to 110 μm. The vertical square solid and half hollow micro pin fins on the combined microchannels were spaced and fixed. The external height of the vertical square solid micro pin fins is h_1 = 70 μm and the external fin hydraulic diameter is a_1 = 40 μm. The optimised design points are expected to meet the manufacturing constraints. The range of Reynolds number of cooling fluid (water) used is in the range $400 \leq R_e W \leq 500$ and for the cool air stream the Reynolds number is $23 \leq R_e A \leq 29$. The thermo-physical properties of water and air used in this study are based on 25°C and the heat flux applied was 2.5×10^6W/m^2. The optimization was performed to find the missing geometry that minimises the peak temperature in the microchannel or maximises global thermal conductance.

Presentation and Discussion of Numerical Results

Figure 5 shows the effect of diameter on the peak temperature. The trends show the existence of an optimal design values which minimises the peak temperature in the microchannel. The microchannel with solid square fins performed thermally better than the half hollow square fin. The values that minimise the peak temperature are within the manufacturing limits. The influence of dimensionless hydraulic diameter of the microchannel heat sink is presented in Fig. 6. As the diameter increases, the peak temperature decreases at $R_e W$ = 500. The micro heat sink with Solid Square fins performed better again (Fig. 6). Similarly, Fig. 7 shows the influence of $(d_h)_{opt}$ on the peak temperature of the microchannel. The microchannel with solid square fins has optimised hydraulic diameter $(d_h)_{opt}$ in the vicinity of 0.135 mm, while the microchannel with half hollow fins optimised in the neighbourhood of 0.165 mm, The results show that the solid square fins optimised at lower peak temperature than the half hollow square fins. Figure 8 shows the minimised temperature as a function of Reynolds number of air. As the value of R_e W increases, the minimised temperature decreases. This shows that increasing the water velocity increases the cooling in the cooling channels and in the entire microchannel. Again, it is obvious from the curves that the microchannel with solid square fins cools faster than the half hollow fins. This also means increase in global thermal conductance C_{max} (Fig. 10 and 11). Similarly, in Fig. 9 the influence of Reynolds of air on the minimised peak temperature is illustrated. As the R_e A increases, the $(T_{max})_{\min}$ decreases. Excess heat is removed from the fins by ensuring adequate air flows over the vertical fins.

Figure 10 shows the influence of $R_e W$ on the global thermal conductance. As the fluid velocity increases, C_{max} increases. The performance of the system is enhanced by increasing the Reynolds number of the water.

Figure 11 shows C_{max} as a function of Reynolds number of air. Increase in R_eA, leads to increase in C_{max}. The relationship is a directly proportional one. More cooling is achieved by allowing more air to flow over the vertical micro fins.

Temperature Distribution and Contour

Figure 12 and 13 show the temperature contour and distribution. It is apparent that the temperature decreases more in the combined microchannel with solid square fins when compared with the microchannel with half hollow square fins, as shown in the figures.

DOI: 10.3844/ajeassp.2022.43.50

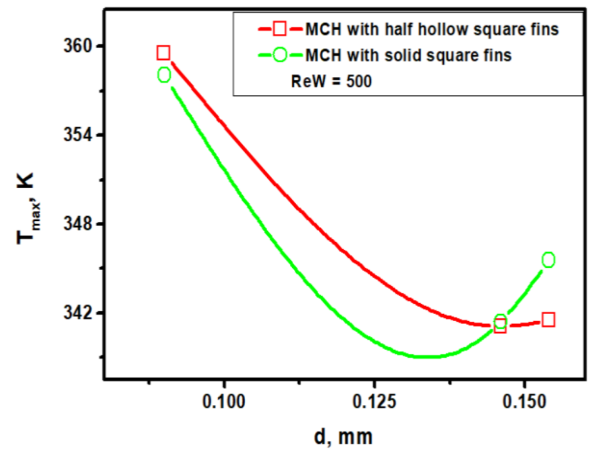


Fig. 5: Shows the influence of hydraulic diameter on the peak temperature

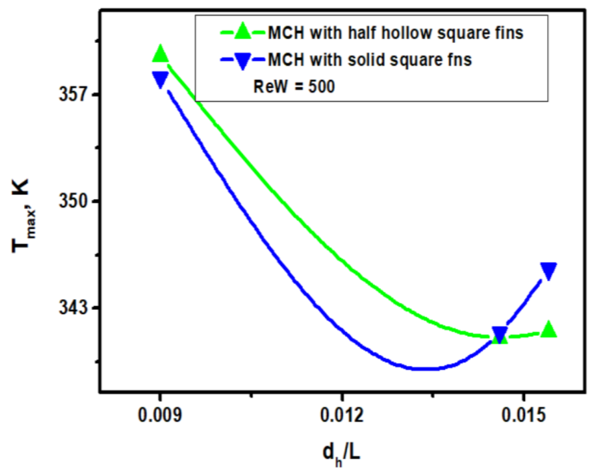


Fig. 6: Shows the influence of dimensionless hydraulic diameter on T_{max}

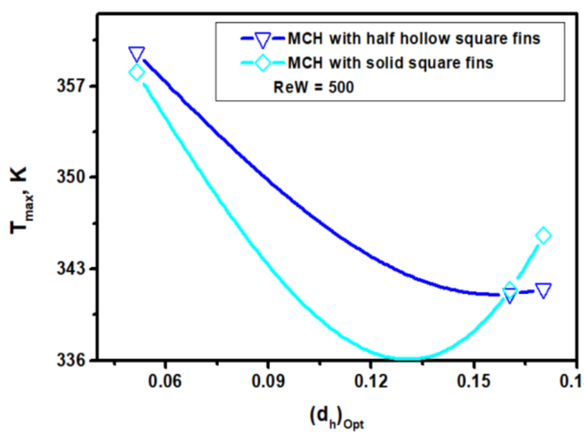


Fig. 7: Shows the influence of optimised hydraulic diameter on T_{max}

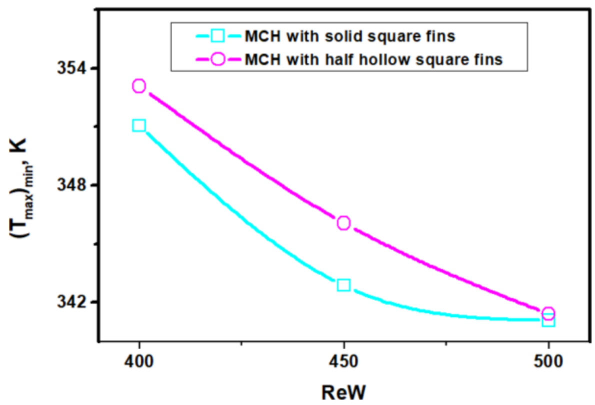


Fig. 8: Shows the influence of Reynolds number of water R_eW on the minimised temperature

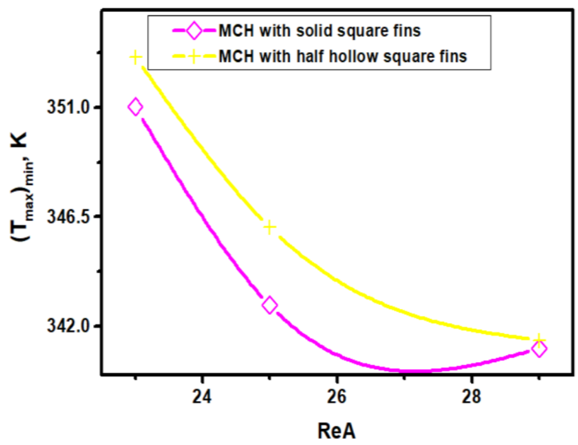


Fig. 9: Shows the influence of Reynolds number of air R_e A on minimised temperature

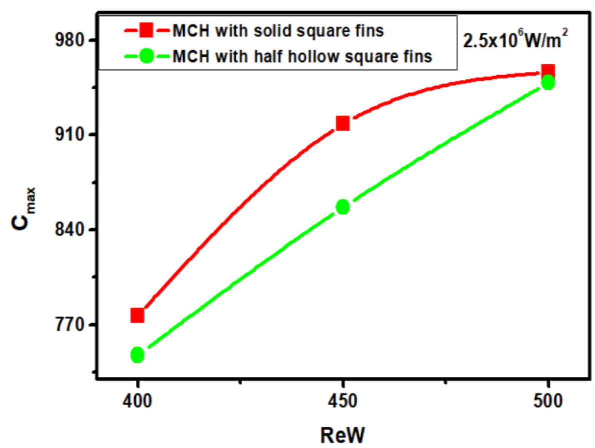


Fig. 10: Shows the influence of Reynolds number of water on the global thermal conductance C_{max}

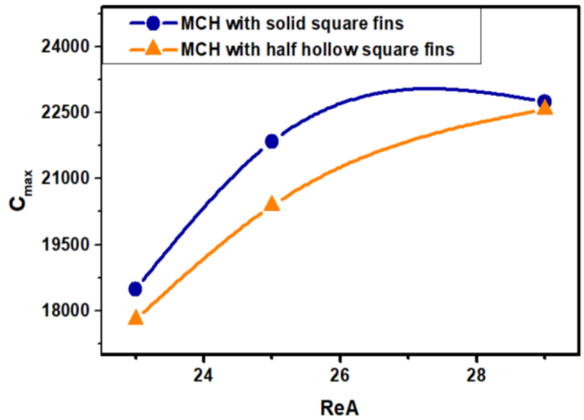


Fig. 11: Shows the influence of Reynolds number of air on C_{max}

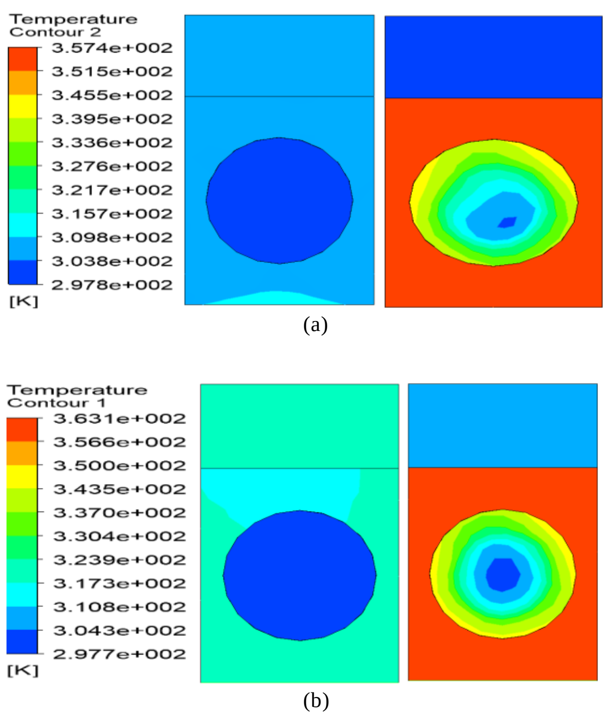


Fig. 12: Shows the inlet and out temperature contour in (a) solid square fins and (b) half hollow square fins

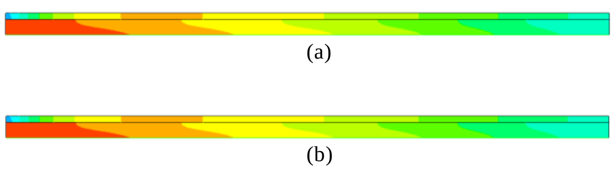


Fig. 13: Show the temperature contour and field of microchannel with (c) solid square fins and (d) half hollow square fins

Table 1: Dimensions of microchannel used for comparison

L (mm)	W (μm)	h (μm)	d_h (μm)	S (μm)
10	200	200	154.9	45.1

Table 1: Dimensions of microchannel used for grid refinement test

N (mm)	M (μm)	H (μm)	d_h (μm)	S (μm)
10	200	254	154.9	45.1

Conclusion

This study presents the numerical optimisation of complex microchannel heat exchangers using constructed theory. The missing geometry that minimises the peak temperature or conversely maximises the global thermal conductance was obtained. The axial length of the microchannel heat sink was 10 mm and the elemental volume V varies from of 0.38 to 0.5 mm^3. The thermo-physical of water and cool air stream were evaluated at 298 K and the range of Reynolds number of water was from 400 to 500, while the air was 23 to 29. The first coolant (water) was used to remove heat at in the bottom of the microchannel with circular channels, but with square micro vertical fins. Water flows through the microchannel and within the inner walls of the vertical fins, in air the second coolants flow over the vertical microchannel externally to remove excess heat in the vertical solid and half hollow fins. The constructed micro fins are light in weight, reduced volume and augment heat transfer, they increased easy access for fluid flow and heat transfer in the system. The numerical result in this study reveals that the microchannel with solid square fins performed better than the half hollow square fins, the author recommends the use of constructed fins for microelectronics cooling.

Acknowledgment

The author wishes to recognise and appreciate the support of Prof. Brandon Collier-Reed, Associate Prof. Melabogo Ngoepe and Prof. Tunde Bello-Ochende, for their professional advice. Others are Engr. Tayo Moshood Tajuddeen (Norway), Mr. James Irlam of UCT, SA and his family, Engr. & Mrs. Tanimu Jatua of UCT and my family members. The Lord bless them all. I am grateful for the connections to the HPC of UCT.

Funding Information

The author sincerely appreciates the financial support of University of Cape Town, South Africa and assistance of my friend, Engr. Tayo Moshood Tajudeen (Norway) towards the success of this research.

Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

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