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Bayesian Models for Time Series with Covariates, Trend, Seasonality, Autoregression and Outliers

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ABSTRACT

Bayesian methods furnish an attractive approach to time series data analysis. This article proposes the forecasting models that can detect trend, seasonality, auto regression and outliers in time series data related to some covariates. Cumulative Weibull distribution functions for trend, dummy variables for seasonality, binary selections for outliers and latent autoregression for autocorrelated time series data are used for the data analysis. The Gibbs sampling, a Markov Chain Monte Carlo (MCMC) algorithm, is used for the parameter estimation. The proposed models are applied to vegetable price time series data in Thailand. According to the RMSE, MAPE and MAE criteria for model comparisons, the proposed models provide the best results compared to the exponential smoothing models, SARIMA models and the Bayesian models with trend, auto regression and outliers.

Keywords: Bayesian Methods, Time Series, Cumulative Weibull Distribution, Trend, Seasonality, Outliers

1. INTRODUCTION

Time series data are observations obtained through repeated measurements over time. For example, measuring product prices each month of the year would comprise time series data. Data collected irregularly or only once are not time series data. Time series data can be decomposed into three main components: trend which is a long term direction, seasonality which is systematic and calendar related movements and irregularity which is unsystematic and short term fluctuations. Some other components, such as outliers and autoregression can also be implicit in time series data. The presence of those components could easily mislead the time series analysis procedure resulting in the wrong conclusion.

A good time series forecast is essential in all fields, such as sciences, industry, agriculture, commerce and economics. The prediction of future events is a critical input into many types of planning and decision making process (Montgomery *et al*., 2008).

Several classical methods have been designed to handle those components. The Holt-Winters exponential smoothing method was first introduced more than half a century ago for the trend and seasonal time series forecast and it is still one of the most popular forecasting systems widely used in many application areas (Szmit and Szmit, 2012). The autoregressive integrated moving average (ARIMA) model is usually used for time series data with trend and autoregression. The seasonal ARIMA denoted as SARIMA is a generalization and extension of the regular ARIMA. It is used for time series where a pattern repeates seasonally over time (Machiwal and Jha, 2012). Besides those methods, Yelland (2010) proposed a Bayesian framework, using binary selections to detect outliers, cummulaitve Weibull distributions to detect trends and latent autoregression to detect the correlated time series data.

Tongkhow and Kantanantha (2012) proposed Bayesian forecasting models by applying and adjusting the model proposed by Yelland (2010) in the way that the distribution of outliers, autoregression and some prior distributions were different. The proposed models were applied to vegetable

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prices in Thailand which were used in the previous study of Tongkhow and Kantanantha (2011) in which multiple regression, ARIMA, exponential smoothing, SARIMA and Bayesian model were used.

This article proposes time series forecasting models using Bayesian methods to detect trend, seasonality, autoregression and outliers. We extend our previous models (Tongkhow and Kantanantha, 2012) by including covariates related to the time series data and seasonality. The Gibbs sampling, one of the most popular Markov Chain Monte Carlo (MCMC) algorithms, is used for parameter estimation. The proposed models are then applied to vegetable price time series data in Thailand which were used in our two previous studies (Tongkhow and Kantanantha, 2011; 2012). The results are compared with the best models in the two previous studies which were exponential smoothing, SARIMA and the Bayesian model with trend, autoregressioon and outliers. For model comparisons, some assessment criteria such as Root Mean Squared Error (RMSE), Mean Absolute Percent Error (MAPE) and Mean Absolute Error (MAE) are employed.

2. MATERIALS AND METHODS

2.1. Exponential Smoothing Models

A simple exponential smoothing model is used to reduce irrigularities in times series data. It assigns exponentially decreasing weights as the observations get older. In other words, recent observations are given relative more weight in forecasting than the older observations (Wang, 2010). A double exponential smoothing model applies the process of a simple exponential smoothing model to account for linear trend in time series data and a triple exponential smoothing model or Holt-Winters model can adjust for both trend and seasonality (Szmit and Szmit, 2012).

2.2. SARIMA or Seasonal ARIMA Model

ARIMA (p, d, q) models (Box *et al*., 1994) take into account historical data and decomposes them into an autoregressive process (AR), an integrated (I) process and moving average (MA) process of the forecast errors. Therefore, the ARIMA models have three model parameters, one for AR(p) process, another one for I(d) process and the other one for MA(q) process.

The seasonal ARIMA model (Machiwal and Jha, 2012) denoted as SARIMA is a generalization and extension of the ordinary ARIMA model to allow seasonality in the data. This seasonal component of the ARIMA model is denoted by capital letters, SARIMA (p, d, q)(P, D, Q)s, where the last bracket indicates the seasonal factor parameters for the order of autoregressive, integration and moving average parts of the model. The first bracket indicates the non-seasonal parameters.

2.3. Bayesian Methods

A hierarchical Bayesian model (Congdon, 2010) is formulated as Equation 1:

$$p(\theta \mid D) = \frac{p(D \mid \theta)p(\theta)}{p(D)} \quad (1)$$

where, $p(D|\theta)$ is a likelihood, $P(\theta|D)$ is a posterior distribution which stands for the marginal probability density of the parameter vector θ given the data D, $p(\theta)$ is a prior distribution of θ, which summarizes any priori or alternative knowledge on the distribution of θ and p(D) is the marginal distribution of data D.

The MCMC algorithms are used for parameter estimation. The MCMC methods provide a way to sample from $P(\theta|D)$ without necessarily knowing its analytic form. The final result of MCMC is a set of vectors θ with density $p(\theta|D)$ in which the model parameters can be estimated. The Gibbs sampling is a common MCMC that can be used for parameter estimation. The most common hierarchical Bayesian model has three stages. A distribution for the data given parameters is specified at the first stage, prior distributions for parameters given hyper-parameters are specified at the second stage and the distribution for hyper-parameters are specified at the third stage. Complicated models can be built through the specification of several simple stages under hierarchical Bayesian models.

2.4. Trend: A Cumulative Wiebull Distribution

A cumulative Weibull distribution for the trend of the new product demand was used by Yelland (2010). It can be applied for product prices since the price varies directly with the demand. The cumulative Weibull distribution is defined as Equation 2:

$$F(x;k,\lambda)\begin{cases} = 1 - e^{-(x/\lambda)^k} & , x \ge 0 \\ = 0 & , x < 0 \end{cases} \quad (2)$$

2.5. Outliers: A Binary Selection

Yelland (2010) adopted a latent binary selection to detect outliers in which the observation Y_{ij} is associated with a latent binary variable $\zeta_{ij} \in \{0, 1\}$ that identifies Y_{ij} as an outlier if $\zeta_{ij} = 1$. The prior distribution for the indicator ζ_{ij} is a Bernoulli distribution such that the probability that $\zeta_{ij} = 1$ is about 5% since the occurrence of outliers is a rare event.

2.6. Latent Autoregression: A(n)

The A(n) (Yelland, 2010) is defined as Equation 3:

$$A_t = \sum_{i=1}^{n} a_i A_{t-1} + \varepsilon_t \tag{3}$$

where, a_i is the latent autoregression coefficient, A_t is the residual variation and n is the order (length) of the latent autoregression. The error term, ε_t, is assumed to follow a normal distribution.

2.7. Seasonality: Dummy Variables

Dummy variables (Wooldridge, 2009) are used to represent seasonality. The regression model with seasonality has the form of Equation 4:

$$Y_t = \sum_{i=1}^{s-1} \omega_i S_{it} + \varepsilon_t \tag{4}$$

where, Y_t is the time series observation and S_{it} is the dummy seasonal variable, s = 4 for quarterly and s = 12 for monthly data. S_{ij} is 1 in their corresponding quarter or month and 0 otherwise. ω_i is the regression coefficient. ε_t is the random error.

2.8. The Proposed Bayesian Model

Let Y_t be time series data at time t, t = 1,…,n. Y_t is assumed normally distributed whose mean can detect trend, autoregression, seasonality and account for some covariates and whose variance can detect outliers. The proposed model is defined as Equation 5:

$$Y_t \sim N\begin{pmatrix} \gamma(\Delta W(t \mid \alpha,\delta) + A_t) + \beta_0 + \sum_{i=1}^{p} \beta_i X_{it} \\ + \sum_{i=1}^{s-1} \omega_i S_{it}, \left[\gamma(1+\zeta_t)\sigma_y\right]^2 \end{pmatrix} \tag{5}$$

The mean of Y_t is Equation 6:

$$E(Y_t) = \begin{pmatrix} \gamma(\Delta W(t \mid \alpha,\delta) + A_t) + \beta_0 + \\ \sum_{i=1}^{p} \beta_i X_{it} + \sum_{i=1}^{s-1} \omega_i S_{it} \end{pmatrix} \tag{6}$$

The variance of Y_t is Equation 7:

$$\mathrm{Var}(Y_t) = \left[\gamma(1+\zeta_t)\sigma_y\right]^2 \tag{7}$$

where, γ is the expectation of Z which is the sum of time series data within the study period. W (t|a,δ) is a cumulative Weibull distribution. A_t is a latent autoregression at time t. ζ_t are outliers at time t. β_0 is an intercept and β_1,.., β_p are regression coefficients corresponding to the covariates X_{1t},…,X_{pt} at time t, respectively. ω_1, ω_2,…, ω_{s-1} are regression coefficients corresponding to the seasonal dummy variables S_{1t},S_{2t},…,$S_{s-1,t}$ at time t, respectively. σ_y^2 is the common variance of Y_t. The prior distributions for Bayesian methods are assigned to each parameter as follows:

$$\beta_0,\beta_1,\ldots,\beta_p \sim N(0,1.0E05)$$
$$\omega_1,\omega_2,\ldots,\omega_{s-1} \sim N(0,1.0E05)$$
$$p(\sigma_y^2) \sim \mathrm{InvGamma}(0.1,0.001)$$

Trend:

$$\Delta W(t \mid \alpha,\delta) = W(t \mid \alpha,\delta) - W(t-1 \mid \alpha,\delta)$$

Where:

$$\alpha \sim N_{[0,\infty)}(\mu_\alpha,\sigma_\alpha^2),\ p(\mu_\alpha) \sim N(0,1.0E05),$$
$$p(\sigma_\alpha^2) \sim \mathrm{InvGamma}(0.1,0.001)$$
$$\delta \sim N_{[0,\infty)}(\mu_\delta,\sigma_\delta^2),\ p(\mu_\delta) \sim N(0,1.0E05),$$
$$p(\sigma_\delta^2) \sim \mathrm{InvGamma}(0.1,0.001)$$

Latent autoregression: AR(1):

$$A_t \sim N(\lambda A_{t-1},\sigma_A^2),\ p(\sigma_A^2) \sim \mathrm{InvGamma}(0.1,0.001)$$
$$\lambda \sim N(0,1.0E05),\ A_0 = 0$$

Outliers:

$$\zeta_t \sim \mathrm{Bern}(0.05)$$

Expectation of total observed data:

$$\gamma \sim N_{[0,\infty)}(\mu_\gamma,\sigma_\gamma^2),\ p(\mu_\gamma) \sim N(0,1.0E05)$$
$$p(\sigma_\gamma^2) \sim \mathrm{InvGamma}(0.1,0.001)$$

Total observed data:

$$Z \sim N(\gamma,\sigma_Z^2),\ p(\sigma_Z^2) \sim \mathrm{InvGamma}(0.1,0.001)$$

2.9. Parameter Estimation: A Gibbs Sampling Algorithm

For parameter estimation, the MCMC algorithm called Gibbs sampling (Bijak, 2011) is used. Sampling from the posterior p(θ|D), θ = (θ_0, θ_1,…, θ_n) the Gibbs sampler requires a random starting point of parameters of interest, $\theta^{(0)}$. For the sampler, there is an initial starting point $\left(\theta_1^{(0)},\theta_2^{(0)},...,\theta_n^{(0)}\right)$.

The steps of Gibbs sampling are:

- Sampling $\theta_1^{(1)}$ from $p(\theta_1|\theta_2^{(0)},...,\theta_n^{(0)},\mathbf{D})$
- Sampling $\theta_2^{(1)}$ from $p(\theta_2|\theta_1^{(1)},\theta_3^{(0)},...,\theta_n^{(0)},\mathbf{D})$. Use updated value of $\theta_1^{(1)}$
- Sampling $\theta_3^{(1)}$ from $p(\theta_2|\theta_1^{(1)},\theta_2^{(1)},...,\theta_n^{(0)},\mathbf{D})$. Use updated value of $\theta_1^{(1)}$ and $\theta_2^{(1)}$
- Similar to the 3 steps above, sample $\theta_4^{(1)},...,\theta_n^{(1)}$
- Sampling $\theta^{(2)}$ using $\theta^{(1)}$ as a starting point and continually using the most updated values
- Repeat until we get M samples, with each sample being a vector of $\theta^{(1)}$, $\theta^{(2)}$,…, $\theta^{(M)}$, where M is the number of samples
- The Monte Carlo Integration can be done on those samples to the quantity of interest. For example, the mean of θ is Equation 8:

$$E(\theta)=\frac{1}{M}\sum_{i=1}^{M}\theta^{(i)} \quad (8)$$

For the proposed model, the likelihood is derived as Equation 9:

$$f(Y_1,...,Y_n|\gamma,w(t),\alpha,\delta,A_1,...,A_n,\beta_0,...,\beta_p,\omega_1,...,\omega_{s-1},\xi_1,...,\xi_n,\sigma_y)=\left\{\prod_{i=1}^{n}f(Y_i|\gamma,w(t),\alpha,\delta,A_1,...,A_n,\beta_0,...,\beta_p,\omega_1,...,\omega_{s-1},\xi_i,\sigma_y)\right\} \quad (9)$$

The prior distributions are:

$$\left\{\begin{array}{l} p(\gamma|\mu_\gamma,\sigma_\gamma^2)p(\mu_\gamma)p(\sigma_\gamma^2)p(w(t)|\alpha,\delta)p(\alpha|\mu_\alpha,\sigma_\alpha^2) \\ p(\mu_\alpha)p(\sigma_\alpha^2)p(\delta|\mu_\delta,\sigma_\delta^2)p(\mu_\delta)p(\sigma_\delta^2) \\ p(A_1,...,A_n|\lambda,\sigma_A^2)p(\lambda)p(\sigma_A^2)p(\beta_0),...,p(\beta_p) \\ p(\omega_1),...,p(\omega_{s-1})p(\xi)p(\sigma_y^2) \end{array}\right\}$$

A posterior distribution is a product of the likelihood and all prior distributions; hence the posterior distribution of the proposed model is Equation 10:

$$p(\gamma,w(t),\alpha,\delta,A_1,...,A_n,\beta_0,...,\beta_p,\omega_1,...\omega_{s-1},\xi_1,...,\xi_n,\sigma_y\mid Y_1,...,Y_n)=\left\{\begin{array}{l} \prod_{i=1}^{n}f(Y_i|\gamma,w(t),\alpha,\delta,A_1,...,A_n,\beta_0,...,\beta_p, \\ \omega_1,...\omega_{s-1},\xi_i,\sigma_y)\{p(\gamma|\mu_\gamma,\sigma_\gamma^2) \\ p(\mu_\gamma)p(\sigma_\gamma^2)p(w(t)|\alpha,\delta)p(\alpha|\mu_\alpha,\sigma_\alpha^2) \\ p(\mu_\alpha)p(\sigma_\alpha^2)p(\delta|\mu_\delta,\sigma_\delta^2)p(\mu_\delta)p(\sigma_\delta^2) \\ p(\lambda)p(A_1,...,A_n|\lambda,\sigma_A^2)p(\beta_0),...,p(\beta_p) \\ p(\xi_1),...,p(\xi_n)p(\omega_1),...,p(\omega_{s-1})p(\sigma_A^2)p(\sigma_y^2)\} \end{array}\right\} \quad (10)$$

Using the Gibbs sampling algorithm, the full conditional distributions need to be explicit. The conditional distribution of each parameter is the product of the likelihood and the all the priors that contain its parameter. An example is Equation 11:

$$p(\gamma|w(t),\alpha,\delta,A_1,...,A_n,\beta_0,...,\beta_p,\omega_1,...\omega_{s-1},\xi_1,...,\xi_n,\sigma_y,Y_1,...,Y_n)=\left\{\begin{array}{l} \prod_{i=1}^{n}f(Y_i|\gamma,w(t),\alpha,\delta,A_1,...,A_n,\beta_0,...,\beta_p, \\ \omega_1,...\omega_{s-1},\xi_i,\sigma_y)p(\gamma|\mu_\gamma,\sigma_\gamma{}^2), \\ p(\mu_\gamma)p(\sigma_\gamma{}^2) \end{array}\right\} \quad (11)$$

2.10. Application

In Thailand, vegetables are high-value economic plants useful for improving income of farmers. The vegetable prices play a major role in coordinating the supply and demand of these products. Hence, the vegetable prices forecast will be advantageous to producers, consumers, processors, rural development planners and other people involved in the vegetable market.

2.11. Data

The data have been extracted from the database of the Office of Agricultural Economics, Ministry of Agriculture and Cooperatives, Thailand (OAE, 2012). The monthly average consumer prices for coriander, green shallot and celery, from 2000 to 2011 (144 months) are used for this study since those vegetables are common and their prices usually fluctuate. For some missing data, simple three-month moving averages of the preceding observations are computed to fill in the missing observations.

2.12. Data Analysis

The proposed Bayesian models were applied to the prices of coriander, green shallot and celery. The Gibbs sampling algorithm was used for parameter estimation via Open BUGS program. The Gibbs sampling was run for 10,000 iterations, discarding the first 1,000 iterations (the burn-in iterations) and the rest was used to compute the posterior means and standard errors. For the model evaluation, simulations were done in R program. The SPSS for Windows was used to estimate the parameters in the exponential smoothing and SARIMA. The RMSE, MAPE and MAE are criteria for the model comparison.

Table 1. Model evaluation for coriander

Parameter	MSE	SE	CIL	CP
γ	0.006	0.007	0.09	0.93
α	0.007	0.005	0.07	0.95
σ_y	0.006	0.007	0.09	0.95
ζ	0.004	0.003	0.06	0.96

Table 2. Model evaluation for green shallot

Parameter	MSE	SE	CIL	CP
γ	0.005	0.008	0.09	0.95
α	0.007	0.005	0.05	0.96
σ_y	0.005	0.006	0.08	0.92
ζ	0.006	0.009	0.09	0.96

Table 3. Model evaluation for celery

Parameter	MSE	SE	CIL	CP
γ	0.009	0.006	0.08	0.92
α	0.008	0.007	0.07	0.95
σ_y	0.007	0.004	0.08	0.93
ζ	0.008	0.006	0.07	0.94

2.13. Model Evaluation

Given that the parameters were obtained from the analysis of each vegetable price data, 500 samples of time series data (Y_t) were generated to evaluate the proposed model. The Mean Squared Errors (MSE), Standard Errors of the MSE (SE) and 95% Confidence Interval Lengths (CIL) of important model parameters are very low, but the 95% Coverage Probabilities (CP) are very high. These results exhibit that the proposed perform very well. The model evaluation for each vegetable is illustrated in **Table 1-3**.

3. RESULTS

The proposed models perform better than the exponential smoothing models, SARIMA models and the Bayesian models with trend, autoregression and outliers in all vegetables since all error measurements of the proposed model are smallest. The graphs of the actual values and the predicted values from the proposed model of each vegetable are shown in **Fig. 1-3**. The error measurements, RMSE, MAPE and MAE, are shown in **Table 4**.

It is evident that the predicted values from the proposed model are very close to the actual ones.

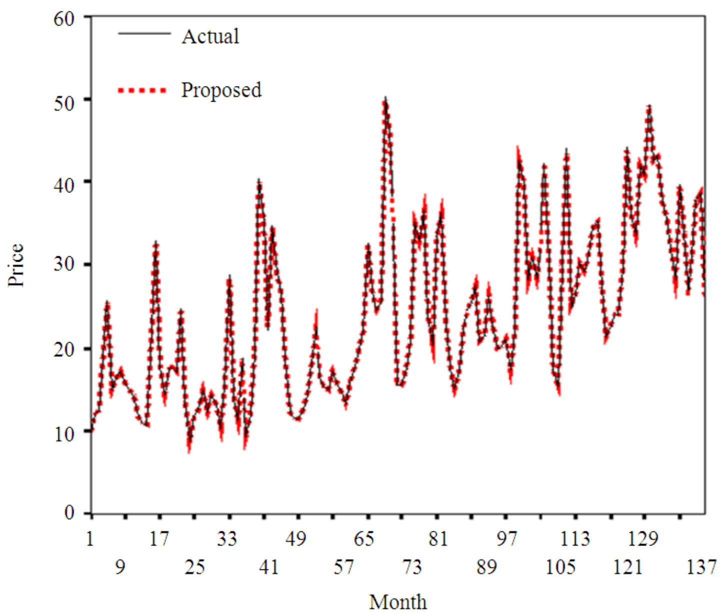

Fig. 1. Actual and predicted prices of coriander

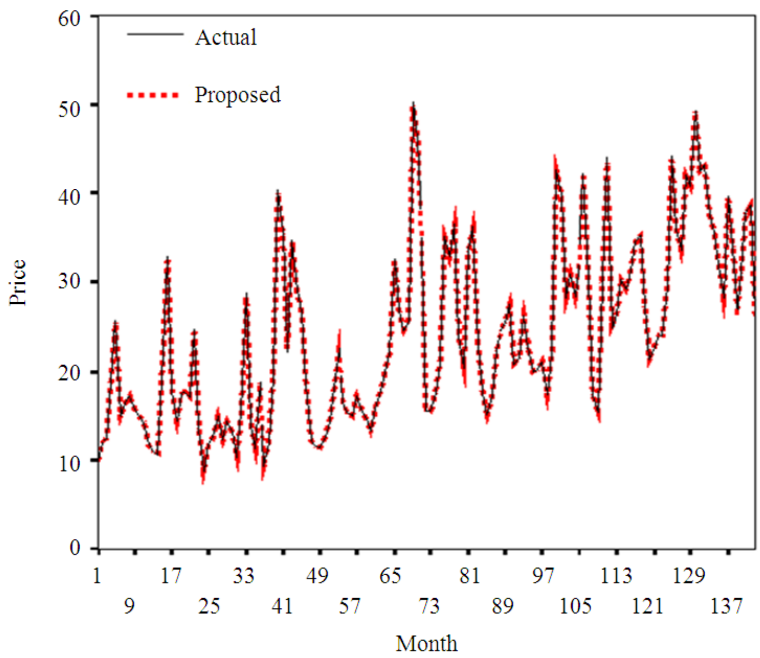

Fig. 2. Actual and predicted prices of green shallot

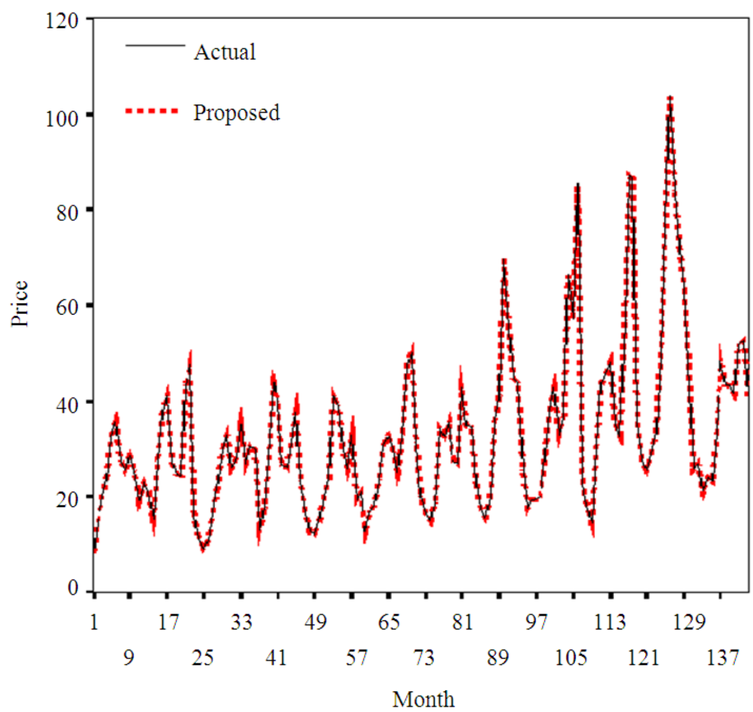

Fig. 3. Actual and predicted prices of celery

Table 4. Model performance comparison

Vetgetable	Model	Error measurement RMSE	MAPE	MAE
Coriander	1. Proposed model	0.32	1.53	0.31
	2. Bayesian model (trend, autoregression and outliers)	0.89	2.88	0.88
	3. Exponential smoothing (simple seasonal)	10.16	20.87	7.44
	4. ARIMA (1,1,3) (1,1,3)s	10.44	25.52	8.27
Green shallot	1. Proposed model	0.42	1.38	0.42
	2. Bayesian model (trend, autoregression and outliers)	1.39	6.54	1.36
	3. Exponential smoothing (Holt-Winters' Additive)	5.94	19.16	4.30
	4. ARIMA (1,1,3) (1,1,3)s	5.95	18.60	4.23
Celery	1. Proposed model	0.35	1.29	0.34
	2. Bayesian model (trend, autoregression and outliers)	1.25	4.66	1.24
	3. Exponential smoothing (simple seasonal)	10.16	22.18	6.93
	4. ARIMA (1,1,3) (1,1,3)s	10.30	24.92	7.24

4. DISCUSSION

The proposed Bayesian models with covariates that can detect trend, seasonality, autoregression and outliers, are appropriate for all vegetables because the models account for all main components which usually occur in the time series data. In contrast, the exponential smoothing models, SARIMA models and the Bayesian models that can detect only trend, autoregression and outliers in our previous study (Tongkhow and Kantanantha, 2012) account for some of those components, so they are inferior comparing to the proposed models. The conjugate inverse gamma priors are assinged to the variance components of the normal distributions, therefore, the full conditional distributions of those variances are inverse gamma distributions. The conjugate inverse gamma prior is one the prior distributions having been suggested in Bayesian analysis (Jackman, 2009).

5. CONCLUSION

As this study has shown, the proposed Bayesian models with covariates that can detect trend using cummulative Weibull distribution functions, seasonality using dummy variables, autoregression using latent autoregressions and outliers using binary selections, are appropriate for all vegetables. The proposed models can be applied to other vegetables or products that have unusual components in the data. This would be valuable to anyone that would like to forecast from those data.

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