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Literature Reviews

Topological Approaches to Mechatronic Systems: A Review

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Abstract: An interest in the mechanical topology approach is to reduce the mass considerably in order to minimize the costs of manufacturing, transporting and purchasing products. The same interest can be found in mechatronic topological approaches where many disciplines should be fit together, facing therefore many and variant constraints. Distinguish topological approaches are found in the literature. Many are more specific for a single domain than a complex system. Two main classes of topological approaches are structured. The first class is based on a theoretical approach that states the KBR topological graph and the MGS language while topology is primarily structural form. These topological modeling approaches tend to combine two disciplines of a Mechatronic system. The second class is the disciplined approaches that recapitulate the topological approaches of each mechatronic discipline. The topological optimization of the mechanical, electronic and control model is summarized. In the light of the literature, there is a lack of a specific topological method for a mechatronic system that encompasses the structural complexity of complex systems. Since the topology is first and foremost a structural shape, the mechanical topological model, which is structurally based, should constitute the algorithmic foundation by integrating the functional and structural constraints of other disciplines.

Keywords: Topological Modeling, Discipline-Based Approach, Optimization of Structural and Mechatronic System, Complex System

Introduction

Mechatronics combines all fields of mechanical, electronic, automatic and computer engineering. Mechatronic systems have a high level of functional integration and have become increasingly important for industrial applications. They are used in various fields, including power systems, transportation, optical telecommunications and biomedical engineering. It can also be seen that these mechatronic systems are based on the mechanical part. It is on this mechanical part that components of different technologies will be integrated to make complex systems. It is known today that the topology of this mechanical part is much more related to topological optimization which is a mathematical method to find the optimal material distribution in a given volume under constraints (Allaire *et al*., 1996). Its main interest lies in the considerable lightening of the parts studied, which leads to a reduction in the total mass of the part in order to reduce the cost of manufacture, transport and purchase. It would therefore be interesting to examine the topology of this mechanical part after the addition of components of different technologies. Indeed, mass is very important in complex mechatronic systems. Take the case of electric vehicles, for example, which have a range limited by the capacity of their battery. This limit distance would increase if the total mass of the vehicle decreased. This is referred to as topological optimization of the electric vehicle (Childs *et al*., 2019). The multidisciplinary design phase of mechatronic systems is a heavy task that considers the integration of several engineering domains simultaneously (mechanics, automation, electronics and computer science). Therefore, one should have a holistic method that will deal with these different engineering domains simultaneously in the development phase and come up with a solution for the system that will be optimal by considering several disciplines. Therefore, there is an interest in analyzing mechatronic topological approaches which will allow having a complete view of the behavior of mechatronic systems (the exchanges between the different disciplines constituting mechatronics, the different interconnection laws). His approaches will also allow us to present an appropriate method for the topological optimization of Mechatronic systems. In the literature, topology is defined either as a kind of modeling using topological modeling graphs (Chaabane, 2014) or by associating it directly with topological optimization (Allaire *et al*., 1996). The topological approach for modeling mechanical systems has been the subject of several research works and dates back to the year 1942 with the work of Kron who applied a topological approach for modeling electrical networks as well as electromechanical systems. Roth in 1955 validated Kron's work mathematically. Branin in 1966 used the same topological structure to describe the physical quantities of multi-physical systems (Plateaux, 2011). Björke takes up this study by proposing an approach applied to production systems (Plateaux, 2011). Egli in 2000 described a general framework for the specification and manipulation of systems such as mass-spring systems and fluid particles based on topological chains. Shai developed a Combinatorial Representation (CR) based on graph theory and matroid theory and he applied it to different engineering domains in particular for the analysis of bar structures (Chaabane, 2014). Modeling will therefore be defined as a theoretical or physical, abstract and more or less faithful representation of a real-world object with a modeling tool to solve a posed problem (Diagne, 2015).

The application of a topological approach to the optimization of mechanical systems dates back to 1996 with the concept of topological optimization to reduce the mass of mechanical parts in order to reduce a certain number of costs (Allaire *et al*., 1996).

It is, therefore, necessary to focus on Mechatronic topological approaches which are also presented either as modeling or as topological optimization approaches. In this paper, which deals with the review of topological approaches to mechatronic systems, we distinguish topological approaches applicable to mechatronic systems in the literature which we will present.

Topological Modeling Approaches

Presented here as topological modeling from the KBR and MGS (General System Model) topological graphs. These topological modeling graphs are based on using mathematical modeling tools such as graphs, simplicial complexes, cellular complexes and chain and co-chain complexes.

The interest in graphs goes back to the problem known as "Königsberg bridges". This problem was solved by Leonhard Euler in 1736 and is stated as follows: Is it possible, starting from one area of the city, to return to the same area by crossing each of its seven bridges once and only once?.

In general, a graph allows the structure and connections of a complex set to be represented by expressing the relationships between its elements. Graphs are therefore a method of thought that allows a wide variety of problems to be modeled by reducing them to the study of vertices and arcs. Since the 1930s, the theory of graphs has undergone very important developments, both theoretically and in terms of applications: Road networks, communication networks, electrical circuits. The linear graph was extended to the modeling of physical systems and became a branch of mathematics dedicated to the study of the topology of systems (topological graph) (Chaabane, 2014).

Linear graphs offer the possibility to model multidimensional systems by combining topological relationships with the constitutive equations of individual components (Sass, 2004).

Modeling from the KBR Topological Graph

The topological graph named KBR was applied by Plateaux in honor of their creators Kron, Branin and Roth (Chaabane, 2014). This diagram allows obtaining any relations between the elements of a studied system according to the specifications and unknowns of the studied system. Also, it allows the distinction between the topological structure of the studied system and the physics associated with it. Figure 1 shows the principle of the KBR topological graph.

The generic approach to the application of the KBR topological graph consists of associating a first topological structure to the system under study. This topological structure is decomposed into nodes, arcs and meshes. This first structure is also associated with two other connected structures called chain and co-chain complex. In a second step, an algebra is associated with these topological structures, which is, in terms of algebraic topology, the description of the relations of chains and co-chains complexes. These relations allow us to describe the connections, to specify equilibrium and behavior laws. The KBR graph then allows to relate chain and co-chain complexes through behavior laws (Chaabane, 2014). These two types of topological entities are associated with the geometrical and physical parameters related to the behavior of the system to be studied.

Figure 1, [C] represents the branch/node incidence matrix with the following equation: [B] = [C] [N] This matrix allows the passage of variables associated with nodes to variables associated with branches. [N] and [B] represent the node and branch matrices respectively. In the case of geometric structures, we can associate with [N] and [B] any multi-vector respectively M_N and M_B, verifying the relation of passage through the incidence matrix [C].

Based on the algebraic duality of the parameters, tensorial and metric relations are added through the admittance [Y] or impedance [Z] matrix. These matrices allow expressing the physical relations and the geometrical

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constraints which intervene between the various elements of the graph, i.e. the primal object and the dual object such as displacement/force and intensity/voltage. The existence of the matrix [Z_N] is subject to the condition that the matrix [Y_N] is invertible.

The KBR topological graph can be applied to the following different systems: Mechanical systems (simple and complex), mechatronic systems.

In order to apply KBR topological graph modeling for mechatronic systems, Plateaux proposed an approach based on the Modelica language within the Dymola environment (Plateaux *et al*., 2009). Indeed, the Modelica object-oriented language takes into account topological connections: Two connected objects exchange their variables by writing term-to-term equality. Also, in most of the objects created in Modelica, the relationships are acausal (without any causal link) and therefore the two equations I = U/R and U = RI are equivalent. Indeed, the equation I = U/R does not mean assigning the value U/R to I and the equation U = RI does not mean assigning the value RI to U. Indeed, the topological nature of the MODELICA language is limited to 0- and 1-simplexes (only representation of 1-simplexes) and access to higher dimensions can only be done via transformations into a 0-simplex (Plateaux, 2011). Also, MODELICA associates topology and behavior which limits the generality of the model studied. Also, the MODELICA language does not allow the modeling of dynamic systems with dynamic structure (Plateaux, 2011). Modeling can be done indirectly for specified elements and by describing the changes that can be caused by their topology changes as in the case of a switch for which there are two positions open or closed.

Given the limitations of the MODELICA language, (Chaabane, 2014) adopted another topological approach allowing to dissociate the topology/behavior of the studied system to have a local generic model allowing the optimization of the system behavior according to the global system. As a language allowing the application of this topological approach, he applied the MGS (General Simulation Model) language.

Modeling from the MGS Language

Chaabane (2014) in his thesis entitled Geometric and Mechanical Modelling for Mechatronic Systems defines topology as a unifying basis for modeling. Chaabane starts from the modeling of mechanical systems (three-bar lattices, bar structures, beam structures) to arrive at the modeling of mechatronic systems based on topological approaches. He, therefore, developed topological modeling approachable to dissociate the topology (interconnection law) and the behavior (physics) of the studied system based on the topological graph KBR. The application of this approach is based on the MGS language, an abbreviation for "General System Model". In this section, we will present the two approaches for topological modeling of Mechatronic systems which are the KBR topological graph and the MGS language.

MGS is a research project of IBISC (Laboratory for Computer Science, Integrative Biology and Complex Systems) of the University of Evry (the MGS home page and Cohen, 2004). This project has two complementary objectives. On the one hand, the study and development of the contribution of notions of a topological nature in programming languages and, on the other hand, the application of these notions to the design of new data and control structures for the simulation of dynamically structured systems, in particular in the field of biology and morphogenesis. In addition to basic elements such as scalar constants, variables, functions, control structures, system functions,..., MGS integrates new types of values called topological collections and their transformations (Chaabane, 2014). Topological collections and their transformations provide a uniform framework for specifying and manipulating data structures and they bring a new notion of rewriting.
The MGS language allows:

- Taking into account topological relations in a programming language
- Simplification of the modeling of complex systems with a dynamic structure: The system is described by a set of local interactions between more elementary entities
- The separation of topology and physics: The topological structure is independent of the behavior

A topological collection represents the state of a dynamic system, i.e., the interconnection between its different elements. A transformation represents the evolution of the topological collection, i.e., the law of behavior of the different elements of the system.

In this topological MGS approach, Chaabane was interested in using topological collections to present the topology of the system under study (interconnection law) and transformations to specify its behavior law. The flow principle of the MGS language is shown in Fig. 2.

These topological modeling approaches (KBR graph and MGS language) have been applied only to the modeling of simple and complex mechanical structures including bar structures, beams, lattices, piezoelectric stacks, piezoelectric lattices and single-stage spur gears

The Discipline-Based Approaches

The discipline-based approach is defined on the basis of the topological optimization principle presented by (Casner *et al*., 2011). It groups together the different methods for the topological optimization of systems. Topological optimization methods are nowadays very popular and integrated into several design software. They provide the designer with a tool of choice for obtaining optimized shapes in the design phase (Takougoum, 2018).

Topological optimization is initially based on finite element analysis and is currently used for the optimization of structures or shapes (Allaire, 2003), i.e., in mechanics. However, in the case of systems coupling several physical domains, such as mechatronic systems, it is useful to consider the possibility of extending topological optimization to such systems. This type of optimization allows determining the best possible solution, without any restriction, even if it means changing the system model.

It is realized that mechanical systems are the basis of Mechatronic systems as they represent the structure of the Mechatronic system where one will add to that an electronic board consisting of electronic elements, a control system consisting of sensors, actuators and a data acquisition chain which will be managed by an information program. Optimizing such a system becomes a little more delicate. We can use a topological optimization approach which will lead us to an optimal final system.

The discipline-based approach addresses this concern. It gathers the different methods of topological optimization of systems and is close to the multi-level optimization which is widely used in the literature and consists in performing optimization of different subsystems, representing the disciplines (mechanical, electronic, automatic ...) implemented by the mechatronic system (Fig. 3); before performing a global optimization of the mechatronic system (Coelho and Breitkop, 2009).

Indeed, we often find ourselves in the case where we have separate mechanical and electrical engineering teams, each of which produces models corresponding to their fields of application and without real interaction between them to lead to a mechatronic product combining more or less homogeneously mechanics, electronics and automatics, which tries to correct the problems of interaction. For the topological optimization of mechatronic systems, we will have: A mechanical topological model to be optimized, an electronic topological model to be optimized and a control topological model to be optimized.

The topological optimization of a mechatronic system, as we see it, is a process that allows optimizing, not the parameters of a model, as is the case of parametric optimization, which is widely used in the literature, but the model itself. Optimizing the mechanical model is already possible, but the question is how to optimize the electric model and the automatic model (Casner *et al*., 2011).

The topological optimization of mechanical systems, currently used is based on the principle of finite element modeling (optimization of structures or shapes) (Allaire, 2003) and allows the optimal model to be sought without any restrictions, even if this means making substantial changes to the initial model. But what about electronic and automatic model optimization?

First of all, what is meant by the topological optimization of electrical systems? Let's take a simple example, such as the design of an analog low-pass filter, it would be interesting to have a computer-aided design system that, according to the defined criteria and constraints, would propose the optimal circuit that allows realizing this filtering (Casner *et al*., 2011).

Similarly, for the control structure, it is possible to choose between several controls structures, such as PI or PID. The objective of the topological optimization algorithm is thus to determine the best corrector among the set of control structures based on stability and robustness criteria (rejection of external disturbances) (Casner *et al*., 2011).

The diagram presented in Fig. 4 summarizes concisely what is expected from the topological optimization of mechatronic systems with possible use of multi-level optimal design aspects.

It can be seen that the topological optimization of mechatronic systems is firstly achieved by optimizing its mechanical structure using one of the different mechanical topological optimization methods (homogenization, BESO, level-set...).

Electronic topological optimization using optimal circuit design tools. Control comes next. Figure 5 shows the principle of mechatronic topological optimization in detail.

It would be important to know the different methods of topological optimization of systems (mechanical, electronic and control).

Table 1: Summary of parameters used in the SIMP method (Takougoum, 2018)

Setting	Designation	Value
dg	Mesh size	250
f	Volume fraction	5%
p	Penalty coefficient	3
$r_{\min}$	Filter radius	$1.25 \times \delta$
ρ_{vide}	Lower limit of density	0.001
m	Limit of density variation between 2 iterations	0.2
η	Damping coefficient	0.5
k	Empirical coefficient amplifying the effect of dis (e, v)	1
Δ	Convergence criterion of the SIMP method	0.5 %

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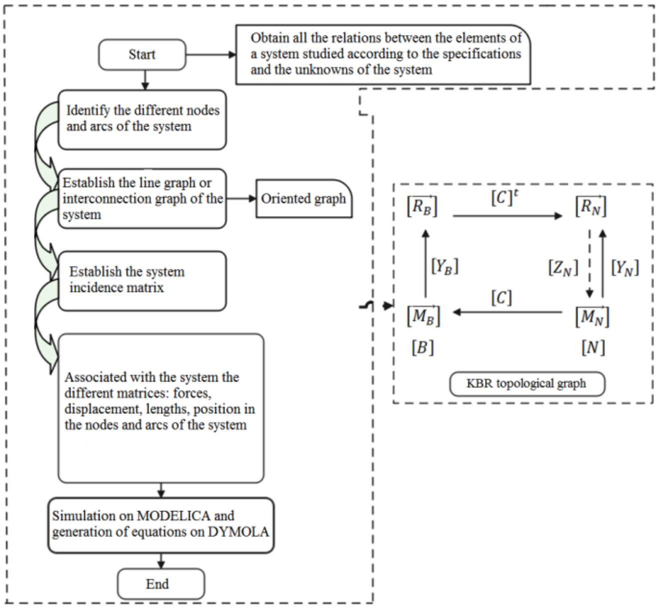

Fig. 1: Principle of the KBR topological graph

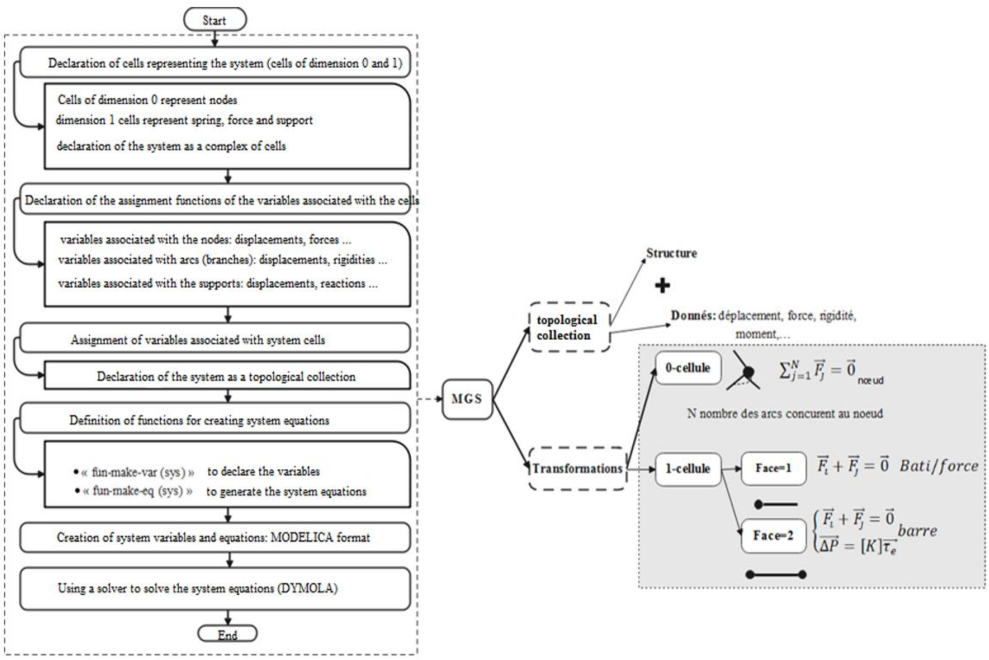

Fig. 2: MGS language flow principle

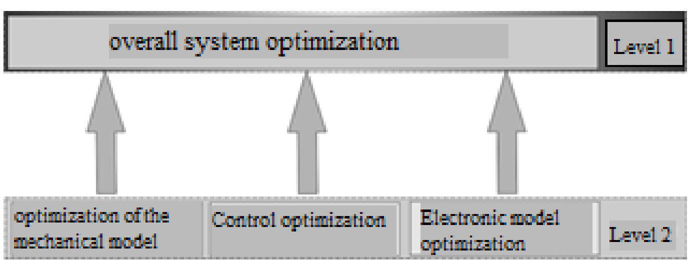

Fig. 3: Possible multilevel optimization methodology (Coelho and Breitkop, 2009)

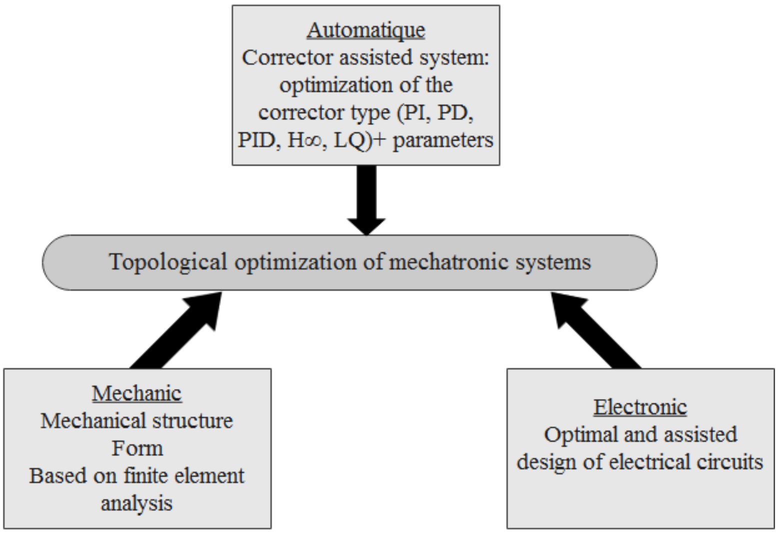

Fig. 4: Principles of topological optimization of mechatronic systems (Casner *et al*., 2011)

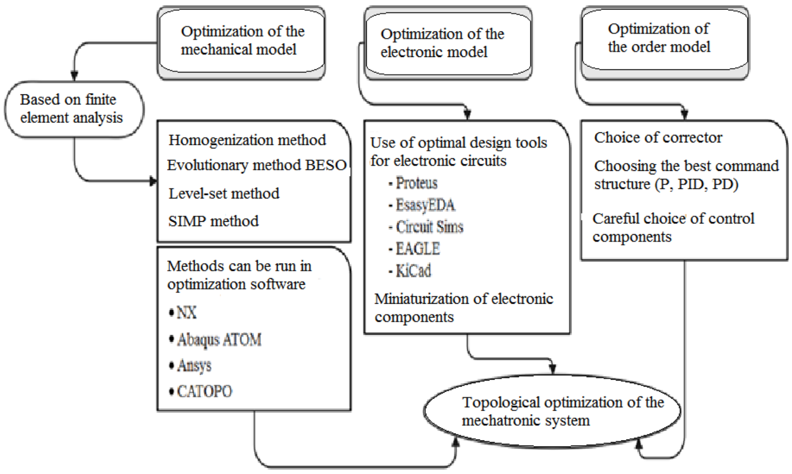

Fig. 5: Principle of topological optimization of a mechatronic system from the disciplined approach

Topological Optimization Methods for Mechanical Systems

The topology of a part refers to the nature and connectivity of the elements of the domain occupied by the material and, as a corollary, the number, shape, dimensions and location of the perforations of said domain. Topological optimization is a method of optimization that consists of searching for the silhouette of a structure subject to certain constraints without any preconceived ideas about its shape, its dimensions, the number of its constituent elements and their connectivity. The nature (shape, type, dimensions) of the components present in the optimized shape is suggested by the result of the optimization process without any restrictions or prior specification (Takougoum, 2018).

General Formulation of an Optimization Problem

In an optimization problem, one or more objective functions are minimized (maximized) while respecting a number of constraints. Optimization methods are generally based on a significant mathematical foundation, regardless of the field of application (logistics, finance, mechanics, etc.) (Craveur *et al*., 2014). An optimization problem most commonly presents itself in the following form:

$$\begin{aligned}&\min_{xi} f(x)\\&\text{Under stress } g_j(x) \le g_j^{\max},\ j = 1,\ldots,m\\&h_k(x) = 0,\ k = 1,\ldots,l\\&\min_{xi} \le xi \le xi^{\max}\end{aligned} \tag{1}$$

where, x_i is the design variables, f is the objective function, g_j (x) and h_k (x) are the inequality and equality

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constraints respectively, m is the mass of the system to be optimized, l is the dimension (length or width). In practice, the functions involved in the optimization problem are the mass, the displacements, the first of the eigen frequencies, the local constraints of Von Mises.

In mechanics, the most common goal of topological optimization is to reduce the weight of a part while retaining a maximum of stiffness, i.e., conserving the material only where it is most essential. The objective of mass or volume to keep is set by the user and is therefore expressed as a constraint while the objective function consists in minimizing compliance. The latter corresponds to the displacements under imposed loadings. There is also the possibility of minimizing the volume (or the mass) under constraints of imposed displacements (Childs *et al*., 2019).

In structural mechanics, analytical solutions do not exist, or only for simple cases. For the resolution of industrial problems, we have recourse to the finite element method. Optimization is therefore based on digital approaches and requires significant (IT) resources. The optimum is reached once no progress on the objective function is possible without violating a constraint (Craveur *et al*., 2014).

The most classic formulation of a topological optimization problem is therefore the next:

$$\begin{aligned} &\min_{\mu} C \\ &\textit{under stress}\; K_q^{\mu} = g \qquad (2) \\ &V \le V_{\max} \end{aligned}$$

where, C is the compliance, K is the global stiffness matrix, q is the vector of nodal displacements, g is the vector of applied displacements and $C = g^T q v = \sum_{i=1}^{n} \mu_i V_i$ the volume, where n is the number of elements and μi and V_i are respectively the pseudo-density and the volume of the element i considered and min means minimize.

In practice, the pseudo-density of element i, μ_i, is defined as: $p_i = \mu_i.p_0$

Or ρ_0 corresponds to the density of the reference material used and where ρ_i therefore corresponds to the effective density of the element i (Childs *et al*., 2019).

Topological optimization methods can be grouped into four main families (Bendsoe and Sigmund, 2003): Homogenization methods, revolutionary or evolutionary, SIMP (Solid Isotropic Material with Penalization), by level lines (or level-Set).

Homogenization Method

The optimization of mechanical structures is a very important field from the point of view of applications, which has recently seen many advances. Besides the classical boundary variation methods (which date back at least to Hadamard; see for instance (Murat and Simon, 1976; Pironneau, 1984) a new optimization method has appeared, called topological, based on the homogenization theory. The latter method has a very low computational cost because it captures shapes on a fixed mesh, but it is mainly restricted to linearized elasticity (Allaire *et al.*, 2002).

Homogenization aims to approximate materials with very heterogeneous microscopic properties, such as composites, with macroscopically homogeneous material. The principle of homogenization methods starts from the simple observation that a single hole in a part makes it more fragile than a set of tiny holes distributed throughout the structure for the same mass.

The idea is to introduce a set of small holes into the initial material from the start, thereby changing its microstructure. The subsequent optimization problem is to find the best possible use of the material by including these composites. One of the major difficulties in the formulation is the need to carefully model the perforated materials. The macroscopic properties of the latter are calculated using the homogenization method or theory and the holes (the void) are modeled by very low-density materials. Homogenization in this case becomes intrinsic to the optimization method (Takougoum, 2018). An optimized result using the homogenization method is Fig. 6.

Evolutionary Method

The BESO method is a finite element-based topology optimization method in which inefficient material is iteratively removed from a structure while efficient material is simultaneously added to the structure. Compared to ESO, which is limited to material removal, the BESO method is much more efficient. This method was first proposed by (Querin *et al*., 1998) to improve the optimization results and speed of the ESO algorithm. Later, a modified version of the BESO algorithm was presented by (Huang and Xie, 2007) to solve the non-convergence and mesh dependency problems associated with earlier BESO algorithms.

(Tang *et al.*, 2015) proposes a modified BESO algorithm to optimize the thickness of a mechanical structure (lattice braces). Instead of directly deleting or adding elements as in the conventional BESO method, the thickness of each strut is updated during the optimization iteration to redistribute the material in the design space. This optimization is designed to emulate the process of bone remodeling known as Wolff's law (Frost, 1994). The material will be removed in low-stress areas and formed in high-stress areas. The volume of material removed is equal to the material added, thus keeping the total volume unchanged. A detailed description of the main steps of the proposed optimization algorithm is given as follows (Fig. 7).

Figure 8 and 9 shows us the optimized engine mount (Tang *et al*., 2015).

Although the BESO method gives a binary result based on a computation of the stresses (unlike for example the method of homogenization), the quality of the result depends on the size of the meshes which must be small (fine mesh). On the other hand, a judicious choice of the variables of the method is essential. Indeed, the ratios used depend on certain parameters such as the counter of the states of equilibrium during the process, as well as the coefficient and the number of oscillations (when an element is added several times and then deleted after successive iterations). An exhaustive comparison of these methods with the SIMP method can be found in (Takougoum, 2018).

Level-Set Method

The Level-set method focuses mainly on the displacement of the border (edges of the shape), that is to say the material-void interface. The model is first of all discretized and the initial topology on which the optimization will be based is chosen. The speed of movement of the boundary is then calculated by differentiating the objective function from the shape. This speed is used to move the boundary during the optimization process.

Although the homogenization and level-set methods have similar costs and results, the latter is very sensitive to the initial model (number of holes) and easily converges towards local minima. On the other hand, in the case of complex objective functions or in nonlinear elasticity, a combination of the two methods is suggested, considering the result of the homogenization method as the initial model before application of the level-set method (Allaire, 2004; Van Dijk *et al*., 2013).

SIMP Method

The SIMP method, for Solid Isotropic Material with Penalization, is one of the most documented topological optimization methods in the literature. She consists of a distribution of the density of matter in the elements of the mesh of a model under certain constraints in order to identify the most rigid form. In doing so, it distributes a given amount of vacuum in the model. This porosity of the material is measured using the material density (x, y, z) which varies from 0 to 1, where 0 represents the absolute vacuum (absence of material) and 1 the solid material (absence of porosity).

Figure 10 presents the algorithm of the SIMP method which will be detailed in the rest of this section.

Modeling and Mechanical Conditions

Whether for a new product or for a pre-existing product, the application of the SIMP method begins with modeling in CAD software. Model amounts to describing in a simplified way a system or problem using, in particular, a physical, geometric, or mathematical representation called a model. So in general, the more precise the modeling, the closer the simulations will be to the actual behavior of the product under consideration. A good approximation of the real shape, particularly of the non-design area, is important. After obtaining the initial geometric model, we apply the conditions mechanical which include the boundary conditions, the loadings and the characteristics of the material used. In our test case, the SIMP method will have for objective a minimization of the compliance, in other words, a maximization of the total rigidity of the model considered, subject to a volume constraint.

Figure 11 shows a test case model of the gateway type used by way of illustration for presenting all of the methodologies developed. The dimensions of the footbridge overall 5.6 m × 5 m × 25 m. The figure also shows the conditions for limits in displacements imposed at the level of the abutments and the loading in the form of pressure evenly distributed in the bay. Loading is $PY = -10$ kN/m^2. The Young's modulus of the material used is $E = 69$ GPa and the Poisson's ratio of $v = 0.3$. The volume fraction conserved is $f = 5\%$ (Takougoum, 2018).

Subdivision into Design and Non-Design Sub-Areas

Once the problem is modeled and the mechanical conditions (including boundary conditions, loadings and characteristics of the material) are applied, one specifies the sub-fields of design and those of non-design. This subdivision of the initial model is essential in order to guarantee the functionality of the structure after optimization. The design sub-domain is the space that will be optimized, in other words in which the material will be optimally distributed. The non-design subdomain is the space that will not be affected by the optimization process, or the space containing the functional places of the model. In most cases, the boundary and loading conditions are applied in the non-design subdomain. The initial domain and the design and non-design subdomains are defined as a B-REP model for Boundary REP resentation (Takougoum, 2018). The latter represents a solid by its borders (in the form of a set of surfaces called shells) and allows a precise definition of its interior, its exterior and the border that separates them (Cuillière *et al*., 2013). A B-REP model has two types of information: Topology (volume, faces, edges and vertices) and geometry (surfaces containing the curves delimiting the faces and containing the lines, then the points delimiting the contours and forming the vertices). Provided that these surfaces are closed, orientable, bounded, connected and do not intersect, the solid is represented by the union of its faces.

Figure 11 presents an illustration of the result of the subdivision of the model case test in design (in blue) and non-design (in red) subdomains.

Mesh

The close similarity between the B-REP model and the data structure of a mesh (tetrahedron, triangle, edge and node) is the cornerstone of the mesh generation process. The mesh of a domain (Frey and George, 1999) is the spatial discretization of the latter using well-defined finite elements, called meshes. The mesh step is compulsory and preliminary to the finite element analysis. In an unstructured automatic mesh context, in addition to being generated automatically, the mesh must meet the specifications of the design and non-design subdomains, mainly to ensure continuity and compliance at the interface between the two subdomains (Cuillière *et al*., 2013). During the mesh, the tetrahedral finite elements are tagged as belonging either to the design subdomain or to that of non-design. In practice, we define the entire model, then the non-design model. The design subdomain is automatically obtained implicitly, such as subtracting non-design from the entire model. A mesh method based on the frontal method (Frey and George, 1999) had already been developed and implemented in our work environment. Readers are encouraged to read the references (Cuillière *et al*., 2013; Frey and George, 1999) for more details.

Figure 12 illustrates the result of this procedure on the test case model. It is about a uniform discretization by using a constant nodal deviation *dg* = 250 mm. The design part is in blue and the non-design part is in red (Takougoum, 2018).

Formulation of the Problem as a Distribution of Matter

Let us consider a mechanical element to be optimized, in particular the model case test. The latter occupies a volume$V$in space and is subdivided into subdomains. Consequently to this subdivision, the porosity will be redistributed only in the design domain and the non-design domain will have a fixed density value equal to 1, that is, to say absence of vacuum. In order to achieve the optimal distribution of the material in the design domain, each element of the mesh is assigned a different fictitious material according to its density. To avoid the problems of numerical instability in practice one uses $0<\rho vide$ = 0.001. The definition of the virtual Young modulus $E\,(x,y,z)$ materials related to density distribution is done according to the law of penalization $p(x,v,z)^p$ (Takougoum, 2018):

$$E\,(x,y,z)=E_0\times p\,(x,y,z)^p \tag{3}$$

where, E_0 is the actual Young's modulus of the material and p is the coefficient of penalization. The effect of the penalization is to make the contribution to the overall stiffness of the intermediate density elements more cost-effective in order to avoid the formation of microstructures within the *design* domain. The relaxation of the optimization problem, which allows for a continuous distribution of the material density between 0 and 1, does not allow for discrete results, hence the importance of the penalization factor p (Takougoum, 2018). As schematized in Fig. 13, the higher the coefficient p is going to be, the more the low-density elements will be attracted towards 0 (the void). The higher density elements will approach 1 and constitute the optimized model. For example, the density ρ = 0.4 becomes 0.4; 0.07 and 0.01 after applying the penalty coefficients 1, 3 and 5 respectively.

The value recommended by (Bendsoe and Sigmund, 2003) during a 3D optimization is $p \geq 3$. The latter proposed equations to find the value of the coefficient p in 2D and 3D (two and three Dimension) according to the Poisson's ratio ν of the material used:

$$p \geq \max\left\{\frac{2}{1-\nu},\frac{4}{1+\nu}\right\} in\ 2D \tag{4}$$

$$p \geq \max\left\{15\times\frac{1-\nu}{7-5\nu},\frac{3}{2}\times\frac{1-\nu}{1-2\nu}\right\} in\ 3D \tag{5}$$

For a given structure, maximizing its overall stiffness amounts to minimizing the overall work of external forces. This study, called compliance, is defined by:

$$\tilde{c}=\{U\}^t\times[K]\times\{U\}\, avec\, \{F\}=[K]\times\{U\} \tag{6}$$

where $\{\tilde{U}\}$ is the total displacement vector, $\{F\}$ the global vector of the applied forces and $[\tilde{K}]$ represents the total stiffness matrix. In the rest of this document, all the affected variables of the hat˜ are those affected by the density (x, y, z). That is $[Ke]$ the expanded local stiffness matrix of an element e of density ρe a mesh made up of Nelements. The modified total stiffness matrix is:

$$[K]=\sum_{e=1}^{N}[K_e]=\sum_{e=1}^{N}(pe)^p\times[K_e] \tag{7}$$

By considering the volume fraction of matter to be preserved $f = \tilde{V}\,/Vd$, or $\tilde{V}$ is the design volume after the vacuum is removed by the relative density distribution and $Vd$the total design volume, the optimization problem comes down to finding the minimum compliance for a fixed quantity of volume to be occupied by the material. It reformulates thus (Bendsoe and Sigmund, 2003):

$$Minimize\ \tilde{c}=\sum_{e=1}^{N}(pe)^p\times[K_e]\times\{U\}^t\times\{U\}$$
$$with: \begin{cases} E\ \ (x,y,z)=E\times p(x,y,z)^p \\ V\ \ \sum_{e=1}^{N} p_e\times v_e = f\times V_d\ \ et\, 0<p_{vide}\leq p\leq 1 \\ \{F\}=[k]\times\{U\} \end{cases} \tag{8}$$

Results of the SIMP Method

Figure 14 shows the material density distribution in the *design* domain at the end of the SIMP optimization process, together with the associated longitudinal sections (Fig. 14a, b,c). The process converged after 24 iterations and the final compliance is $\tilde{C}$ = 523 Joules. The optimized model consists of mesh elements that depend on a noted extraction density. The elements for which $\rho_{seuil}\rho \geq \rho_{seuil}$ are kept and those whose $\rho < \rho_{seuil}$ are disabled because they are considered empty. These are not part of the optimized model. The Fig. 14d,e,f give a good illustration of the shape optimized for different values of the extraction density ρ_{seuil} (Takougoum, 2018).

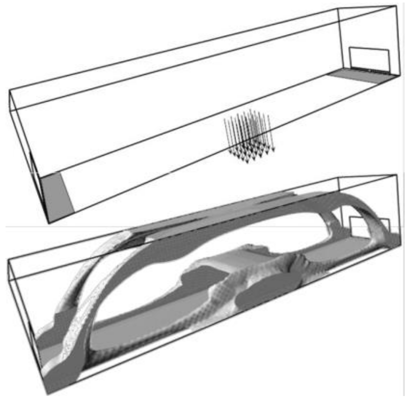

Fig. 6: Result of the homogenization method (Allaire *et al*., 1996)

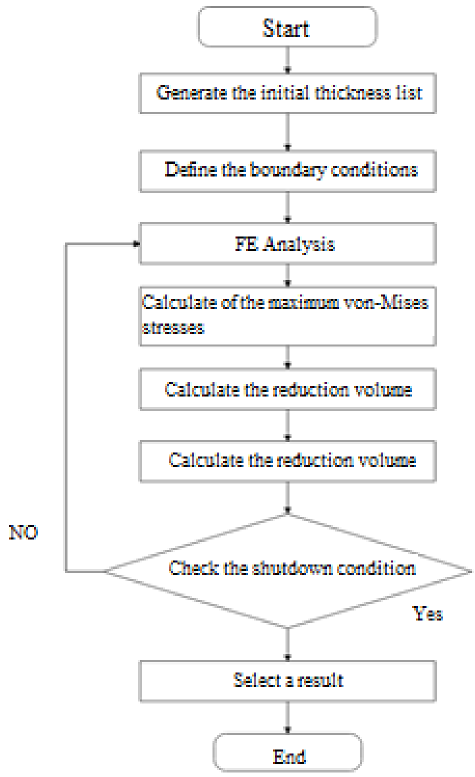

Fig. 7: General workflow of the BESO-based network optimization algorithm (Tang *et al*., 2015)

Fig. 8: (a) An example of an aircraft engine mount (b) Optimized engine mount (Querin *et al*., 1998)

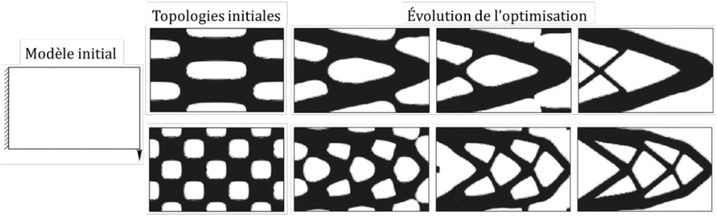

Fig. 9: Topological optimization by the level-set method with 2 topologies

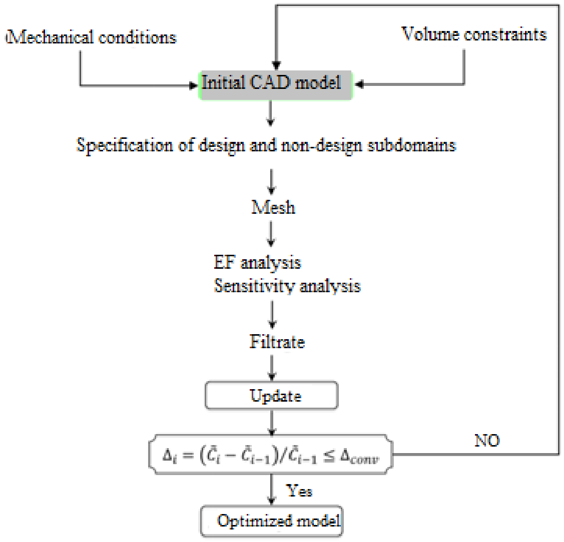

Fig. 10: Diagram of the optimization process using the SIMP method (Takougoum, 2018)

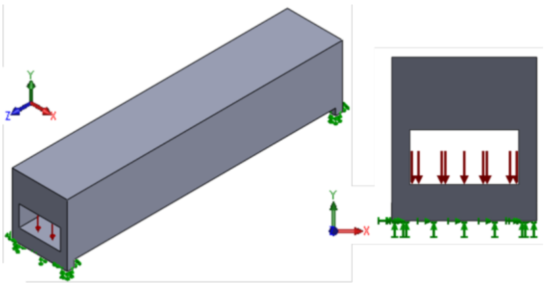

Fig. 11: Illustration of an optimization problem with the model case test (Takougoum, 2018)

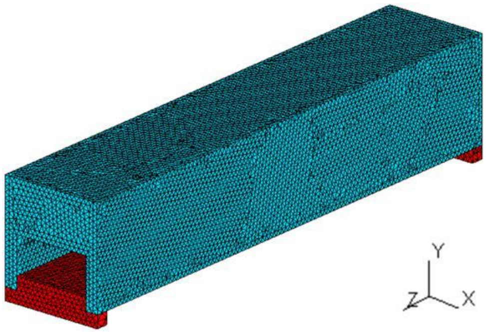

Fig. 12: Illustration of the subdivision of the model case test into design subdomains (in blue) and non-design (in red), with a uniform mesh (dg = 250 mm)

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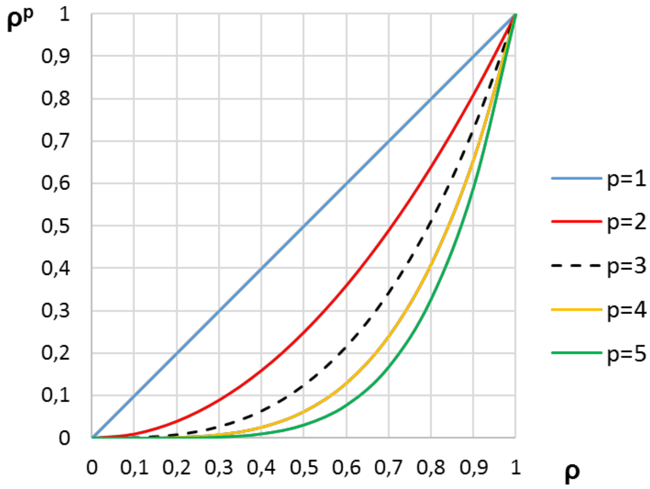

Fig. 13: Effect of the coefficient of penalization *p*.

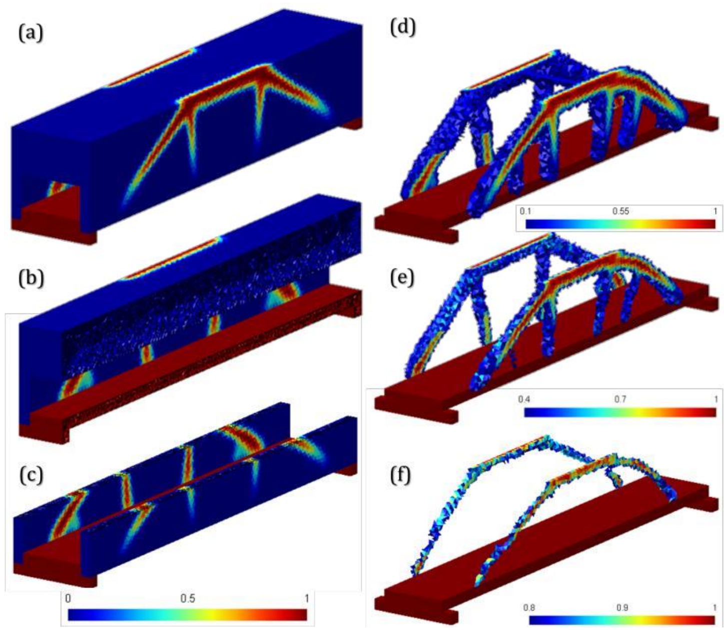

Fig. 14: (a) Result of the SIMP method, (b) and (c) the longitudinal sections of (a), as well as the shapes optimized for ρ_threshold = (d) 0.1, (e) 0.4 and (f) 0.8 (Takougoum, 2018)

We have presented a non-exhaustive number of methods used in the topological optimization of mechanical systems. The SIMP method is one of the most widely used methods for the topological optimization of mechanical systems. This is why much more emphasis has been placed on it. It should be noted here that mechanical topology alone is not sufficient to define Mechatronic topology.

Topological Optimization Method of the Electronic System

An electronic circuit is a set of electronic components interconnected, often by means of a printed circuit board and whose purpose is to perform a function.

Let us first take the example of the topological optimization of electrical systems. When designing an analog low-pass filter, it would be interesting to have a computer-assisted design system that, according to the criteria and constraints defined, would propose the optimal circuit that would allow this filtering to be carried out. We will say here that we have carried out a topological optimization of the low-pass filter (Casner *et al.*, 2011). Topological optimization in electronics can also follow the same logic.

It should also be noted that the miniaturization of electronic components is a very important issue nowadays (Beaulieu, 2019). Miniaturization is the creation of mechanical, optical, or electronic products and their devices at increasingly smaller scales. It allows, among other things: To offer more functionality, reduction of space occupied (items that take up less space are more desirable than items that are increasingly bulky, as they are easier to transport, easier to store and often more convenient to use), weight reduction, price reduction, reduction of energy consumption, reduction of material consumption.

We can therefore conclude by saying that miniaturization solves the problem of topological optimization of electronic systems.

Topological Optimization Method of the Control System

A closed-loop control system consists of the following elements: A sensor to measure the output, a comparator that elaborates the error between the setpoint and the output measurement, a corrector that elaborates the control according to the error signal, a controller that modulates the input signal of the system.

For the optimization of the control structure, it is possible to choose between several controls structures, such as P, PI, or PID. The objective of the topological optimization algorithm is thus to determine the best corrector among the set of control structures based on stability and robustness criteria (Casner *et al.*, 2011).

Choice of Corrector

The structure of the PID corrector shows three actions: The Proportional action, the Integral action and the Derivative action (Tliba *et al.*, 2005).

Action P: Increases bandwidth, thus speed, improves accuracy and degrades stability.

Action I: Slows down the system; improves accuracy by increasing the class of the system and may degrade stability. Also, not very robust to low-frequency disturbances on the setpoint signal.

Action D: Increases system bandwidth and therefore speed, improve stability but amplifies high-frequency measurement noise.

The integral action I has other practical disadvantages. For example, the pure integrator is not technologically feasible because it would mean infinite gain at low frequencies. Similarly, the derivative action D has a similar shortcoming but at high frequencies.

It is not always useful to make a correction with a corrector that combines all these actions. For specific performance improvements, the corrector can be reduced.

P-Corrector improves accuracy, without changing the class of the system.

PI Corrector: Increases the class of the system and therefore improves accuracy.

PID Corrector: Increases the class of the system, thus improving its accuracy, but also speed and stability.

The optimization of the control system will therefore concern the choice of the corrector. A control system can also be optimized in terms of the choice of its constituent elements. For example, let's take the case where we want to choose sensors for our control, it will be beneficial to choose one multifunction sensor rather than several.

Synthesis

There are two main classes of topological approaches applicable to mechatronic systems. One is the topological modeling approach which allows modeling using topological graphs (KBR graph and MGS language). These topological modeling approaches allow obtaining all relationships between the elements of a studied system according to the specifications and unknowns of the system. This modeling also makes it possible to study the behavior of the system, to predict its behavior, to understand its operation, to know the various exchanges between elements of the system. The applications of topological modeling approaches are only for simple and complex mechanical systems (bar structures, beams, lattices, piezoelectric lattices and single-stage spur gears). There is no direct application to mechatronic systems. However, authors such as Chaabane and Plateau have developed methodologies that will allow the application of these modeling approaches to mechatronic systems. It is realized that the topological modeling approach to Mechatronic Systems restricts the definition of topology to modeling. It would be important to extend this definition to topological optimization.

The disciplined approach extends this definition of topology to topological optimization. The discipline approach groups together the different methods of topological optimization of systems (mechanical, electronic, control). It is defined on the basis of the topological optimization principle presented by Casner *et al.* (2011). The disciplined approach allows to optimize separately each sub-discipline (mechanical, electronic, control) constituting the Mechatronic system, to assemble and to obtain the optimized Mechatronic system. The optimization of Mechatronic systems as presented by the disciplined approach just defines concepts for a Mechatronic application. It would be interesting to ask about the assembly of the Mechatronic sub-disciplines once optimized. Because the assembly might be difficult or even impossible.

Conclusion

This article summarizes a review of topological approaches that have to do with a complex system. Having a good shape of a product that integrates different constraints from a complex system is not an easy task. None of the topological approaches in the light of the literature are addressed the issue effectively. On the one hand, the authors topologically model mechatronic systems from the KBR topological graph and the MGS language, while on the other hand, a specific and efficient model for each discipline of the Mechatronic system is addressed. Nevertheless, a model of mechatronic as a discipline is missing. This challenges researchers to define a topological mechatronic approach that should integrate the constraints of each corporation. Since the topology is morphologically structural, one of the tracks would be to adapt the mechanical approach to take into account the structural requirements of other disciplines.

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Author's Contributions

Jean Bosco Samon: Is the main author of this study because he defined the research topic. He designed the research plan, organized the study, contributed to the writing of the manuscript and finally participated in all proofread and reviewed the paper.

Damasse Harold Tchouazong: Is the co-author of this article because he started research as PHD student on the topological complex structure. He contributed to the writing of the manuscript and gathered essential data.

Ethics

I hereby the corresponding author of the manuscript declare that the manuscript titled: Topological Approaches to Mechatronic Systems: A Review, has not been published, that it is not under consideration for publication elsewhere, that its publication is approved by all authors and tacitly or explicitly by the responsible authorities where the work was carried out. It is not stolen or unsheathed from master's theses or doctoral dissertations that are not supervised by the author or of any other research. I take all the legal responsibilities in case this provided information is not correct. I make a sincere effort to ensure the accuracy of the material described herein. No fund has been received for this work. The use of part of the document or all of its content deserves to site the author or to seek his approval. I confirm that I have reviewed and complied with the relevant Instructions to Authors, Ethics in Publishing policy, Declarations of Interest disclosure and information for authors. I am also aware of the publisher's policies with respect to retractions and withdrawal.

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