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A Repairable System with N Failure Modes and K Standby Units Using Separation of Variable Method

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Abstract: This study deals with a repairable system with N failure modes and K standby unit. Using separation of variable method by solving linear ordinary differential equation for system equations obtained by using the supplementary variable method.

Key words: Repairable system, failure mode, standby, separation of variable method, linear ordinary differential equation, supplementary variable method

INTRODUCTION

A repairable system with two failure modes and one standby unit has been analyzed[1,2]. A repairable systems with different condition has been analyzed in[3-5]. Mitsuo[6] studied and analyzed a repairable system with N failure modes and K standby units. He used the laplace transform of the state probability for a system equations obtained by using the supplementary variable method. He could not got the probability that 1st unit is under operational state and the probability that standby unit is under operational state function in time t. The present research studied by another method[6] "separation of variable method" for solving the partial and ordinary differential equation. We get the probability that 1st unit is under operational state and the probability that standby unit is under operational state function in time t. In particular cases we studied the probabilities by using Weibull distribution for nonlinear hazard function and we analysed numerically comparing between Rayleigh distribution (linear hazard function) and Exponential distribution (constant hazard function).

Analysis: The following assumptions are applied to analyze the model.

* The states of the system are represented by its state number and pair (0,0) state 0 and i imply that .1st and (i+1)th units are under operational state. States (0,j) and (i,j) imply that 1st and (i+1)th units are under failed state for failure mode j.
* Operational and standby units are identical.
* Failure rate of failure mode j is constant.
* Rate that the failed unit is replaced with the standby unit is constant .
* The times to repair are generally distributed with repair rate $\eta_j(x)$ from the failed state (0,j) or (i,j)at repair time x .
* The system is state 0 at $t = 0$

Figure 1 shows the state transition diagram for a repairable system with N failure modes and K standby units.

Calculating $P_0(t)$: By using supplementary variable method the following sets of integro-defferential equation can be derived for calculating $P_0(t)$.

$$\frac{dP_0(t)}{dt} + Z(t)P_0(t) = \sum_{j=1}^{N}\int_0^{\infty}\eta_j(x)P_{0,j}(t,x)\,dx \qquad (1)$$

$$\frac{\partial P_{0,j}(t,x)}{\partial t} + \frac{\partial P_{0,j}(t,x)}{\partial x} = -[\eta_j(x)+\alpha]P_{0,j}(t,x) \qquad (2)$$

Where:

$P_0(o) = 1, \quad P_{0,j}(t,0) = \lambda_j P_0(t), \quad P_{0,j}(o,x) = 1$

let $P_{0,j}(t,x) = y$

$\dot{y} + y' = -[\eta_j(x)+\alpha]y$,

Let $y = X(x)T(t)$ and substituting we get:

$$\frac{-\dot{T}}{T} = [\eta_j(x)+\alpha] + \frac{X'}{X} = k \;, \quad k \neq 0$$

By solving this equation by separation of variable method we get:

$T = c_1 e^{-kt}$, $X = c_2 e^{-[\eta_j(x)+\alpha]x}$

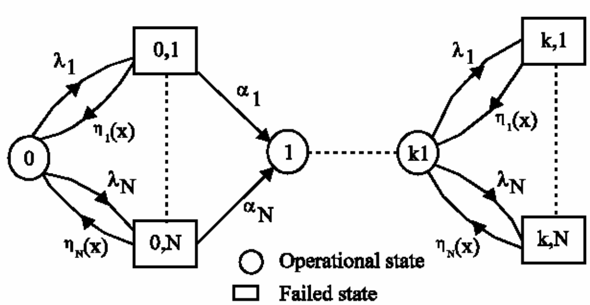

Fig. 1: State transition diagram for a repairable system with N failure modes and K standby units

By using the assumption $P_{0,j}(0,0)=1, P_{0,j}(0,x)=1$ we get:

$$P_{0,j}(t,x)=e^{-[\eta_j(x)+\alpha]t}, \quad (3)$$

Substituting in Eq. 1, we get:

$$\frac{dP_0(t)}{dt}+Z(t)P_0(t)=\sum_{j=1}^{N}\int_0^{\infty}\eta_j(x)e^{-[\eta_j(x)+\alpha]t}dx$$

By solving this linear (O.D E), we get:
The probability that 1st unit is under operational state at time t is:

$$P_0(t)=e^{-\int_0^{\infty}Z(t)dt}\int e^{\int_0^{\infty}Z(t)dt}[\sum_{i=1}^{N}\int_0^{\infty}\eta_i(x)e^{-[\eta_i(x)+\alpha_i]t}dx]dt \quad (4)$$

Calculating $P_i(t)$: By using supplementary variable method the following sets of integro-defferential equation can be derived for calculating $P_i(t)$

$$\frac{dP_i(t)}{dt}+Z(t)P_i(t)=\sum_{j=1}^{N}\int_0^{\infty}\eta_j(x)P_{i,j}(t,x)dx+\alpha\sum_{j=1}^{N}P_{i-1,j}(t) \quad (5)$$

$$\frac{\partial P_{i,j}(t,x)}{\partial t}+\frac{\partial P_{i,j}(t,x)}{\partial x}=-\eta_j(x)P_{i,j}(t,x) \quad (6)$$

Where, $P_i(o)=0$, $P_{i,j}(t,0)=\lambda_j P_i(t)$, $P_{i,j}(o,x)=1$

let $P_{i,j}(t,x)=z$

$\dot{z}+z'=-\eta_j(x)z$

let $z=X(x)T(t)$ and substituting

$$\frac{-\dot{T}}{T}=\eta_j(x)+\frac{X'}{X}=1\ ,\quad 1\neq 0$$

By solving this equation by separation of variable method $T=\beta_1 e^{-lt}$, $X=\beta_2 e^{-[\eta_j(x)-l]x}$

By using the assumption $P_{i,j}(0,0)=1, P_{i-1,j}(0,x)=1$, we get:

$$P_{i,j}(t,x)=e^{-\eta_j(x)t} \quad (7)$$

Substituting in Eq. 5, we get:

$$\frac{dP_i(t)}{dt}+Z(t)P_i(t)=\sum_{j=1}^{N}\int_0^{\infty}\eta_j(x)e^{-\eta_j(x)t}dx+\alpha\sum_{j=1}^{N}P_{i-1,j}(t)$$

By solving this linear (O.D E), we get:
The probability that (i+1)th unit is under operational state at time t is:

$$P_i(t)=e^{-\int_0^{\infty}Z(t)dt}\int e^{\int_0^{\infty}Z(t)dt}[\sum_{j=1}^{N}\int_0^{\infty}\eta_j(x)e^{-\eta_j(x)t}dx]dt+\alpha\sum_{j=1}^{N}P_{i-1,j}(t) \quad (8)$$

Particular Cases: Using Weibull distribution we have the nonlinear hazard function take the form $Z(t)=\lambda\ t^n$
The probability that 1st unit is under operational state at time t is:

$$P_0(t)=e^{-\int_0^{\infty}\lambda t^n dt}\int e^{\int_0^{\infty}\lambda t^n dt}[\sum_{j=1}^{N}\int_0^{\infty}\eta_j(x)e^{-[\eta_j(x)+\alpha]t}dx]dt \quad (9)$$

The probability that (i+1)th unit is under operational state at time t is:

$$P_i(t)=e^{-\int_0^{\infty}\lambda t^n dt}\int e^{\int_0^{\infty}\lambda t^n dt}[\sum_{j=1}^{N}\int_0^{\infty}\eta_j(x)e^{-\eta_j(x)t}dx]dt+\alpha\sum_{j=1}^{N}P_{i-1,j}(t) \quad (10)$$

When n=1, the distribution becomes Rayleigh distribution (linear hazard function), we get, the probability that 1st unit is under operational state at time t is:

$$P_0(t)=e^{-\frac{\lambda t^2}{2}}\sqrt{\frac{\pi}{2\lambda}}[\sum_{j=1}^{N}\eta_j(x)e^{-\frac{1}{2\lambda}(\eta_j(x)+\alpha)^2}] \quad (11)$$

When n=0 the distribution becomes Exponential distribution (constant hazard function) we get, the probability that 1st unit is under operational state at time t is:

$$P_0(t)=e^{-\lambda t}\int e^{\lambda t}[\sum_{j=1}^{N}\int_0^{\infty}\eta_j(x)e^{-[\eta_j(x)+\alpha]t}dx]dt \quad (12)$$

When n=1 the distribution becomes Rayleigh distribution (linear hazard function), and we get, the probability that (i+1)th unit is under operational state at time t is:

$$P_i(t)=e^{-\frac{\lambda t^2}{2}}[\alpha\sum_{j=1}^{N}P_{i-1,j}(t)+\sqrt{\frac{\pi}{2\lambda}}\sum_{j=1}^{N}\eta_j(x)e^{-\frac{1}{2\lambda}(\eta_j(x))^2}] \quad (13)$$

When n=0 the distribution becomes Exponential distribution (constant hazard function), we get the probability that (i+1)th unit is under operational state at time t is:

$$P_i(t) = e^{-\lambda t}\int e^{\lambda t}[\sum_{j=1}^{N}\int_0^{\infty}\eta_j(x)e^{-\eta_j(x)t}dx]dt + \alpha\sum_{j=1}^{N}P_{i-1,j}(t) \quad (14)$$

When the system with two failure modes, let N=2, $0 \le x \le 1$

From Eq. 11, we get:

$$P_0(t) = e^{-\frac{\lambda t^2}{2}}\sqrt{\frac{\pi}{2\lambda}}[\eta_1 e^{-\frac{1}{2\lambda}(\eta_1+\alpha)^2} + \eta_2 e^{-\frac{1}{2\lambda}(\eta_2+\alpha)^2} \quad (15)$$

From Eq. 13, we get:

$$P_i(t) = e^{-\frac{\lambda t^2}{2}}[\alpha e^{-[\eta_2+\alpha]t} + \alpha e^{-\eta_2 t} + \sqrt{\frac{\pi}{2\lambda}}(\eta_1 e^{-\frac{\eta_1^2}{2\lambda}} + \eta_2 e^{-\frac{\eta_2^2}{2\lambda}})] \quad (16)$$

From Eq. 12, we get:

$$P_0(t) = \frac{\eta_1}{\lambda-\eta_1-\alpha}e^{-(\eta_1+\alpha)t} + \frac{\eta_2}{\lambda-\eta_2-\alpha}e^{-(\eta_2+\alpha)t} \quad (17)$$

From Eq. 14, we get:

$$P_i(t) = \frac{\eta_1}{\lambda-\eta_1}e^{-\eta_1 t} + \frac{\eta_2}{\lambda-\eta_2}e^{-\eta_2 t} + \alpha e^{\alpha}(e^{\eta_1 t} + e^{\eta_2 t}) \quad (18)$$

Graphical Representation: In particular cases, taking the different values for time t when λ=0.5, α=0.2, $\eta_1 = \eta_2 = 0.1$ 1 the following results are given in Table 1.

Table 1: Represent the relation between the both probability of the first unit and the probability of standby unit with rayleigh distribution and exponential distribution

	Rayleigh dis.		Exponential dis.	
T(h)	$P_o(t)$	$P_i(t)$	$P_o(t)$	$P_i(t)$
0.1	0.3231	0.7612	0.9704	0.9072
0.2	0.3207	0.7679	0.9418	0.9148
0.3	0.3167	0.7713	0.9139	0.9229
0.4	0.3112	0.7873	0.8869	0.9314
0.5	0.3043	0.7912	0.8607	0.9403
0.6	0.2960	0.7986	0.8353	0.9498
0.7	0.2866	0.8115	0.8106	0.9597
0.8	0.2760	0.8256	0.7866	0.9701
0.9	0.2645	0.8301	0.7634	0.9809
1	0.2523	0.8394	0.7408	0.9924

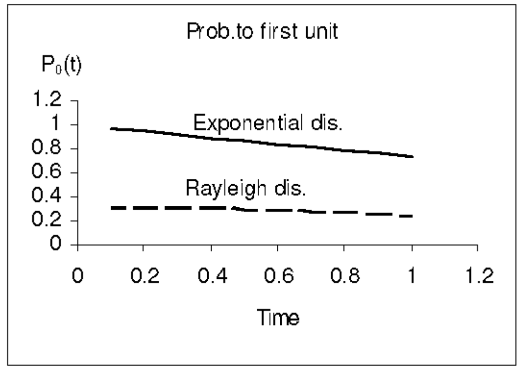

Fig. 2: Relation between the Time and Probability to the First Unit

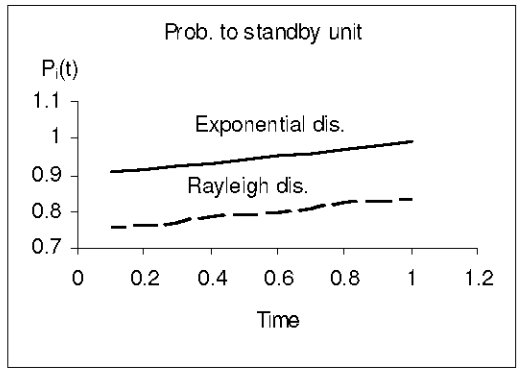

Fig. 3: Relation between the Time and Probability to the Standby Unit

CONCLUSION

Under general assumptions, one have derived results for a repairable system with N failure modes and K standby unit using separation of variable method and showed that our results in particular cases, we note that the probability that 1st unit is under operational state $P_0(t)$ is inversely proportional to time t and the probability that standby unit is under operational state $P_i(t)$ is directly proportional to time t under a constant failure rate and constant replacement rate . In Exponential distribution, the values of $P_o(t)$ and $P_i(t)$ greater than its value with Rayleigh distribution. It could be concluded that the system with Exponential distribution is more a vailable than the system with Rayleigh distribution.

Notation

N number of failure modes

K number of standby units

λ_j failure (hazard) rate of failure mode j, $\lambda = \sum_{j=1}^{N}\lambda_j$

x time since repair began

$g_j(t), \eta_j(x)$ pdf and repair (hazard) rate of failure mode i at repair time x

$Z(t)$ hazard function

α replacement rate

$P_0(t)$ probability that 1st unit is under operational state at time t

$P_i(t)$ probability that (i+1)th unit is under operational state at time t

$P_{0,j}(t,x)$ probability density (with respect to x) that 1st unit is state (0 , j) under repair at time t and its elapsed repair time is x.

$P_{i,j}(t,x)$ probability density (with respect to x) that (i+1)th unit is state (i , j) under repair at time t and its elapsed repair time is x.

$P_{i,j}(t)$ $P_{i,j}(t) = \int_0^\infty P_{i,j}(t,x)dx$ for $i = 1,2,......,K$ and $j = 1,2,.......,N$

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