Journal of Mathematics and Statistics 3 (3) : 100-105, 2007
ISSN 1549-3644

Study of Families of Curves in the Euclidian Plan

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Abstract: Non-standard analysis techniques are more considered in approaching complex mathematical domains. By using some concepts of non-standard analysis methods such as regionalization method, we deal with a family of curves in an Euclidian plan. The solutions of the algebraic equations representing these curves in a plan have an hyperbolic forms.

Key words: Non-standard analysis, regionalization, unlimited number, infinitesimal, appreciable

INTRODUCTION

Our recent work deals with a family of curves in the Euclidian plan by using some concepts of non-standard analysis given by Robinson, A.[1] and axiomatized by Nelson, E.[2]. More precisely, under some conditions concerning domains we show that the solutions of the algebraic curves have geometrical forms (hyperbolic).
we start our study with the algebraic curve

$E(m,n,a)$ defined in $\Re_+^{*2}$ by the set $E(m,n,a)=\{(x,y)\in\Re_+^{*2} / \left(\frac{1}{x}\right)^{2m}+\left(\frac{1}{y}\right)^{2n}=a, \text{m}\geq\text{n}\succ 0, \text{a}>0\ \}$

where (x,y) verify the following equation $x^{2m}y^{2n}a=y^{2n}+x^{2m}$,a>0 real x>0 ,y>0 by using the regionalization method[3].

This curve allows us to define two sets $Q\left(a^{-\frac{1}{2m}},a^{-\frac{1}{2n}}\right)$ and $Q\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$ such that $Q\left(a^{-\frac{1}{2m}},a^{-\frac{1}{2n}}\right)$ the quadrant defined by $x\geq a^{-\frac{1}{2m}}$ *and* $y\geq a^{-\frac{1}{2n}}$ and the vertex $\left(a^{-\frac{1}{2m}},a^{-\frac{1}{2n}}\right)$ and $Q\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$ the quadrant defined by $x\geq\left(\frac{a}{2}\right)^{-\frac{1}{2m}}$, $y\geq\left(\frac{a}{2}\right)^{-\frac{1}{2n}}$ and the vertex $\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$; which allow us to cover the curve $E(m,n,a)$.

From the equation $\left(\frac{1}{x}\right)^{2m}+\left(\frac{1}{y}\right)^{2n}=a$ which defines the curve $E(m,n,a)$ and we can write the function $f_{m,n,a}$ defined from $\left]a^{-\frac{1}{2m}},+\infty\right[$ into $\Re$, such that

$$f_{m,n,a}(x)=\frac{x^{\frac{m}{n}}}{\left(ax^{2m}-1\right)^{\frac{1}{2n}}}$$

Proposition 1: The function $f_{m,n,a}$ has the following properties:

1°) $f_{m,n,a}(x)$ is strictly decreasing on $\left]a^{-\frac{1}{2m}},+\infty\right[$.

2°) $f_{m,n,a}(x)$ has $y=a^{-\frac{1}{2n}}$ as horizontal asymptote

3°) $f_{m,n,a}(x)$ has $x=a^{-\frac{1}{2m}}$ as vertical asymptote

Proof of the proposition 1: We show that $f_{m,n,a}(x)$ is strictly decreasing ; we study the sign of its derivable form:

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Given: $$f'_{m,n,a}(x) = \frac{-\frac{m}{n}x^{\frac{m}{n}}}{x\left(ax^{2m}-1\right)^{\frac{1}{2n}+1}}$$

Since x belongs $\left]a^{-\frac{1}{2m}},+\infty\right[$ is equivalent to

$x \succ a^{-\frac{1}{2m}}$ equivalent $x^{2m} \succ \frac{1}{a}$

Since $-\frac{m}{n} \langle 0$ then $\frac{-\frac{m}{n}x^{\frac{m}{n}}}{x\left(ax^{2m}-1\right)^{\frac{1}{2n}+1}} \langle 0$

However $x\left(ax^{2m}-1\right)^{\frac{1}{2n+1}} \rangle 0$ then

$\frac{x^{\frac{m}{n}}}{x\left(ax^{2m}-1\right)^{\frac{1}{2n}+1}} \rangle 0$ where $f' \prec 0$ then the function f is decreasing.

2- We are going to verify that f has an horizontal asymptote, for this we compute

$$\lim_{x\to\infty} f_{m,n,a}(x) = \lim_{x\to\infty}\frac{x^{\frac{m}{n}}}{\left(ax^{2m}-1\right)^{\frac{1}{2n}}}$$

$$= \lim_{x\to\infty}\frac{x^{\frac{m}{n}}}{a^{\frac{1}{2n}}\,x^{\frac{m}{n}}} = a^{-\frac{1}{2n}}$$

Then $y=a^{-\frac{1}{2n}}$ is an horizontal asymptote.

3- We are going to verify that f has a vertical asymptote for this we compute

$$\lim_{x\to a^{-\frac{1}{2m}}} f_{m,n,a}(x) = \lim_{x\to a^{-\frac{1}{2m}}}\frac{x^{\frac{m}{n}}}{\left(ax^{2m}-1\right)^{\frac{1}{2n}}} = +\infty$$

Then $x = a^{-\frac{1}{2m}}$ is a vertical asymptote.

Lemma d'encadrement : We have the relations :

$$\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right) \in E(m,n,a) \subset Q\left(a^{-\frac{1}{2m}},a^{-\frac{1}{2n}}\right) - Q^0\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$$

with Q^o the interior part of Q.

Proof of the lemma d'encadrement: The vertex $Q\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$ belongs to $E(m,n,a)$

because $\left[\frac{1}{\left(\frac{a}{2}\right)^{-\frac{1}{2m}}}\right]^{2m} + \left[\frac{1}{\left(\frac{a}{2}\right)^{-\frac{1}{2n}}}\right]^{2n} = \frac{a}{2}+\frac{a}{2} = a$ However

$\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right) \in E(m,n,a)$

$$(x_0,y_0) \in Q^0\left[a^{-\frac{1}{2m}}\ ,\ a^{-\frac{1}{2n}}\right] \subset Q\left[a^{-\frac{1}{2m}}\ ,\ a^{-\frac{1}{2n}}\right],$$

following the definition of an interior of a set.

(ii) we show that $E(m,n,a) \Rightarrow \left(\frac{1}{x_0}\right)^{2m} + \left(\frac{1}{y_0}\right)^{2n} = a$

It remains to be shown that

$$(x_0,y_0) \notin Q^0\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$$

If $(x_0,y_0) \notin Q\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$ then

$$(x_0,y_0) \notin Q^0\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$$

By contradiction we suppose that $(x_0,y_0) \in Q^0 \subset Q\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right) - \left\{\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right\}$

If we take a point $(x_0,y_0) \in Q^0 \subset Q\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right) - \left\{\left(\frac{a}{2}\right)^{-\frac{1}{2m}},\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right\}$

Then

$$x_0 \rangle \left(\frac{a}{2}\right)^{-\frac{1}{2m}}\ and\ y_0 \ge \left(\frac{a}{2}\right)^{-\frac{1}{2n}}\ or\ x_0 \ge \left(\frac{a}{2}\right)^{-\frac{1}{2m}}\ and\ y_0 \rangle \left(\frac{a}{2}\right)^{-\frac{1}{2n}}$$

Imply $(x_0,y_0) \notin E(m,n,a)$, hence contradiction.

Lemma of general framing: We have the following relations:

$$\left(\left(\frac{a}{k}\right)^{-\frac{1}{2m}},\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right)\in E(m,n,a)\subset Q\left[a^{-\frac{1}{2m}},\ a^{-\frac{1}{2n}}\right]-Q^{0}\left[\left(\frac{a}{k}\right)^{-\frac{1}{2m}},\ \left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right]$$

Proof of the lemma of general framing:

Let $(x_0\ ,y_0)\in E(m,n,a)\Rightarrow\left(\frac{1}{x_0}\right)^{2m}+\left(\frac{1}{y_0}\right)^{2n}=a$

$$\left(\frac{1}{x_0}\right)^{2m}\rangle a\ \ and\left(\frac{1}{y_0}\right)^{2n}\rangle a\ \ then\,x_0\rangle a^{-\frac{1}{2m}}\ and y_0\rangle a^{-\frac{1}{2n}}$$

Imply $(x_0\,,y_0)\in Q^0\left(a^{-\frac{1}{2m}}\,,a^{-\frac{1}{2n}}\right)\subset Q\left(a^{-\frac{1}{2m}}\ ,\ a^{-\frac{1}{2n}}\right)$

It remains to be shown that $(x_0\,,y_0)\notin Q^0\left[\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right]$

By contradiction we suppose

$$(x_0\,,y_0)\in Q^0\subset Q\left[\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right]-\left\{\left(\frac{a}{k}\right)^{-\frac{1}{2m}},\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right\}.$$

if we take a point

$$(x_0\,,y_0)\in Q^0\left[\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right]-\left\{\left(\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right)\right\}$$

$$x_0\ \rangle\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\quad and\quad y_0\geq\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}$$

then $(x_0\ ,y_0)\notin E(m,n,a)$

where $x_0\ \geq\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\quad and\quad y_0\ \rangle\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}$

Contradiction

Proposition 2: When $m\geq n\rangle 0$ are integers the geometric place of the vertex of the quadrants

$Q\left(a^{-\frac{1}{2m}}\,,a^{-\frac{1}{2n}}\right)$ and $Q\left[\left(\frac{a}{2}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right]$ is the curve of equation $y=x^{\frac{m}{n}}$

Proof of the proposition 2: Since the vertex of the quadrants

$Q\left(a^{-\frac{1}{2m}}\ ,\ a^{-\frac{1}{2n}}\right)$ and $Q\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$

are $\left(a^{-\frac{1}{2m}}\ ,\ a^{-\frac{1}{2n}}\right)$ and $\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$

the writing $a^{-\frac{1}{2n}}=\left(a^{-\frac{1}{2m}}\right)^{\frac{m}{n}}$ and $\left(\frac{a}{2}\right)^{-\frac{1}{2n}}=\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}}\right)^{\frac{m}{n}}$

shows that the vertex verify the equation $y=x^{\frac{m}{n}}$.

Proposition 3: When $m\geq n\rangle 0$ are fixed integers and a fixed real $a=2x_0^{-2m}=2y_0^{-2n}$ the geometric place of the vertex of the quadrants

$Q\left(a^{-\frac{1}{2m}}\ ,\ a^{-\frac{1}{2n}}\right)$ and (resp $Q\left(\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right)$

is the curve of equation $y=x^{\frac{m}{n}}$ (resp $y=(k-1)^{-\frac{1}{2n}}\ x^{\frac{m}{n}}$) when a ranges over $]0\ ,\ +\infty[$

Proof of the proposition 3: The vertex of the quadrants

$Q\left(a^{-\frac{1}{2m}},\ a^{-\frac{1}{2n}}\right)$ and $Q\left(\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right)$

are

$S\left(a^{-\frac{1}{2m}}\ ,\ a^{-\frac{1}{2n}}\right)$ and $S\left(\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\ ,\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right)$.

The writing $a^{-\frac{1}{2n}}=\left(a^{-\frac{1}{2m}}\right)^{\frac{m}{n}}$ shows that the coordinates of $S\left(a^{-\frac{1}{2m}}\ ,\ a^{-\frac{1}{2n}}\right)$. Verify the equation $y=x^{\frac{m}{n}}$.

The writing $\left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}} = (k-1)^{-\frac{1}{2n}}\left(\left(\frac{a}{k}\right)^{-\frac{1}{2m}}\right)^{\frac{m}{n}}$

shows that the coordinates of $S\left(\left(\frac{a}{k}\right)^{-\frac{1}{2m}}, \left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right)$ Verify the equation $y=(k-1)^{-\frac{1}{2n}} x^{\frac{m}{n}}$

Reciprocaly: A point (x_0, y_0) of the curve $y=(k-1)^{-\frac{1}{2n}} x^{\frac{m}{n}}$ is the vertex of the quadrant $Q\left(\left(\frac{a}{k}\right)^{-\frac{1}{2m}}, \left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right) = Q\left(\left(x_0^{-2m}\right)^{-\frac{1}{2m}}, \left(y_0^{-2n}\right)^{-\frac{1}{2n}}\right)$.

$$Q\left(\left(k\frac{x_0^{-2m}}{k}\right)^{-\frac{1}{2m}}, \left(\frac{(k-1)}{k}.\frac{k}{(k-1)}y_0^{-2n}\right)^{-\frac{1}{2n}}\right) = Q\left(\left(\frac{a}{k}\right)^{-\frac{1}{2m}}, \left(\frac{a(k-1)}{k}\right)^{-\frac{1}{2n}}\right)$$

From the equality $k\frac{x_0^{-2m}}{k} = \frac{a}{k}$ *we obtain* $a = k.x_0^{-2m}$

From the equality $\frac{(k-1)}{k}.\frac{k}{(k-1)}y_0^{-2n} = \frac{k-1}{k}a$ *we obtain* $a = \frac{k}{k-1}y_0^{-2n}$

which give us:

$a = k.x_0^{-2m} = \frac{k}{k-1}y_0^{-2n}$; hence the point (x_0, y_0) is a curve point.

Parameters monitoring the shape of the curves:

Let $V = \left(\frac{a}{2}\right)^{-\frac{1}{2n}} - a^{-\frac{1}{2n}} = \left(1-2^{-\frac{1}{2n}}\right)\left(\frac{a}{2}\right)^{-\frac{1}{2n}}$

Vertical thickness of the « main band of encadrement ».

Let $h = \left(\frac{a}{2}\right)^{-\frac{1}{2m}} - a^{-\frac{1}{2m}} = \left(1-2^{-\frac{1}{2m}}\right)\left(\frac{a}{2}\right)^{-\frac{1}{2m}}$

Horizontal thickness of the «main band of encadrement ».

Let $r = \frac{m}{n}$ Parameter monitoring the curve $C(m,n)$ of the vertex of the quadrants.

$Q\left(a^{-\frac{1}{2m}}, a^{-\frac{1}{2n}}\right)$ *and* $Q\left(\left(\frac{a}{2}\right)^{-\frac{1}{2m}}, \left(\frac{a}{2}\right)^{-\frac{1}{2n}}\right)$.

And let $C(m,n)$: The curve of equation $y = x^{\frac{m}{n}}$

Comparaison of the thickness: We have two situations

(i) $\frac{m}{n} \approx 1$ the $C(m,n)$ curve has the shape of the right-line $y = x$

(ii) $1 \prec\prec \frac{m}{n} \prec\prec \infty$ the $C(m,n)$ curve has the shape of the right-line $y = x^{\left(\frac{m}{n}\right)^o}$

Proposition 4: If $n \succ 0$ is infinitely big ,then the vertical thickness V is substantially positive if and only if : $a \in \left(\frac{A_+}{2n}\right)^{2n}$

Proof of proposition 4: We show that $V = \left(1-2^{-\frac{1}{2n}}\right)\left(\frac{a}{2}\right)^{-\frac{1}{2n}} \in A_+$ *is equivalent to* $a \in \left(\frac{A_+}{2n}\right)^{2n}$

Indeed:

$\left(1-2^{-\frac{1}{2n}}\right)\left(\frac{a}{2}\right)^{-\frac{1}{2n}} \in A_+$ *is equivalent to* $\left(\frac{a}{2}\right)^{-\frac{1}{2n}} \in \left(\frac{A_+}{1-2^{-\frac{1}{2n}}}\right)$

We apply the limited development of $2^{-\frac{1}{2n}} = e^{-\frac{1}{2n}\log 2}$

$$2^{-\frac{1}{2n}} = e^{-\frac{1}{2n}\log 2} =$$

$$= 1 - \frac{1}{2n}\log 2 + \frac{1}{2!}\left(\frac{1}{2n}\log 2\right)^2 - \frac{1}{3!}\left(\frac{1}{2n}\log 2\right)^3 + \ldots.$$

$$= 1 - \frac{1}{2n}\log 2\left[1 - \frac{1}{2!}(\frac{1}{2n}\log 2) + \frac{1}{3!}\left(\frac{1}{2n}\log 2\right)^2 + \ldots..\right]$$

we take $\gamma = 1 - \frac{1}{2!}(\frac{1}{2n}\log 2) + \frac{1}{3!}\left(\frac{1}{2n}\log 2\right)^2 + \ldots.$

$\gamma \approx 1$. If n>0 is infinitely big, then $2^{-\frac{1}{2n}} = 1 - \left(\frac{1}{2n}\log 2\right)\gamma \approx 1 - \frac{1}{2n}\log 2$

Therefore $\left(\frac{a}{2}\right)^{-\frac{1}{2n}} \in \frac{A_+}{1 - 1 + \frac{1}{2n}\log 2} = \frac{A_+}{\frac{1}{2n}\log 2}$

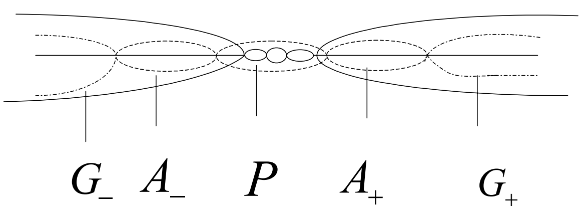
$$\frac{a}{2} \in \left(\frac{A_+}{\frac{1}{2n}\log 2}\right)^{-2n}$$

is equivalent to $\frac{a}{2} \in \left(\frac{2nA_+}{\log 2}\right)^{-2n}$ is equivalent to $\frac{a}{2} \in \left(\frac{\log 2}{2nA_+}\right)^{2n}$

As $\frac{\log 2}{A_+} \approx A_+$ therefore $a \in \left(\frac{A_+}{2n}\right)^{2n}$ c.q.e.d

Thus $\left(\frac{P}{2n}\right)^{2n}$, $\left(\frac{G_+}{2n}\right)^{2n}$ Are the complements of $\left(\frac{A_+}{2n}\right)^{2n}$

The graphical representation:



Proposition 5: If $m \succ 0$ is infinitely big, then the horizontal thickness h is substantial positive if and only if $a \in \left(\frac{A_+}{2n}\right)$

Proof of proposition 5: The same proof as for the proposition 4 by substituting m for n.

Proposition 6: If $m = n\delta$, $\delta \in 1 + \frac{L}{\ln 2n}$ then we have $\left(\frac{A_+}{2m}\right)^{2m} = \left(\frac{A_+}{2n}\right)^{2n}$

with L : limit , A_+: substantial positive.

Study of the tangents: Let $E(m,n,a)$ a family of curves of the plan $\Re^2$ defined by the equation $\left(\frac{1}{x}\right)^{2m} + \left(\frac{1}{y}\right)^{2n} = a$

($m \geq n \geq 0$) infinitely big integer, a>0 real. the tangent in point (x_0, y_0) of the $E(m,n,a)$ curve has the equation $n(y - y_0)x_0^{2m+1} + m(x - x_0)y_0^{2n+1} = 0.$ Indeed: the equation of the tangent in a point (x_0, y_0) is:

$$y - y_0 = f'_{m,n,a}(x_0)(x - x_0) \quad (*)$$

Where $y = f_{m,n,a}(x) = \frac{x^{\frac{m}{n}}}{(ax^{2m} - 1)^{\frac{1}{2n}}}$

then $f'_{m,n,a}(x) = \frac{-\frac{m}{n}x^{\frac{m}{n}-1}}{(ax^{2m} - 1)^{\frac{1}{2n}+1}}$

And since $y^{2n} = \frac{x^{2m}}{ax^{2m} - 1}$, we replace in (*) we obtain $y - y_0 = \frac{-\frac{m}{n}x_0^{\frac{m}{n}-1}}{(ax_0^{2m} - 1)^{\frac{1}{2n}+1}}(x - x_0)$

hence

$$(y - y_0)(ax_0^{2m} - 1) = \frac{-\frac{m}{n}x_0^{\frac{m}{n}}}{x_0(ax_0^{2m} - 1)^{\frac{1}{2n}}}(x - x_0)$$

$$(y - y_0)\frac{x_0^{2m}}{y_0^{2n}} = -\frac{m}{n}\frac{y_0}{x_0}(x - x_0)$$

imply $(y - y_0)x_0^{2m+1} = -\frac{m}{n}y_0^{2n+1}(x - x_0)$

$$\boxed{n(y - y_0)x_0^{2m+1} + my_0^{2n+1}(x - x_0) = 0}$$

as $x_0 > 0, y_0 > 0$ we have the equation:

$$y - y_0 + \frac{m}{n}\frac{y_0^{2n+1}}{x_0^{2m+1}}(x - x_0) = 0$$

Situation where the slope is infinitely small:

$$\frac{m}{n}\frac{y_0^{2n+1}}{x_0^{2m+1}} \in P \text{ is equivalent } y_0^{2n+1} \in \frac{n}{m} x_0^{2m+1} P$$

$$\text{is equivalent } y_0 \in \left(\frac{n}{m} P\right)^{\frac{1}{2n+1}} x_0^{\frac{2m+1}{2n+1}}$$

Result 1: the slope is infinitely small $(x_0, y_0) \in \Re_+^{*2}$ as $y_0 \in \left(\frac{n}{m} P\right)^{\frac{1}{2n+1}} x_0^{\frac{2m+1}{2n+1}}$

Situation where the slope is substantial positive:

$$\frac{m}{n}\frac{y_0^{2n+1}}{x_0^{2m+1}} \in A_+ \text{ is equivalent } y_0 \in \left(\frac{n}{m} A_+\right)^{\frac{1}{2n+1}} x_0^{\frac{2m+1}{2n+1}}$$

if the slope is appreciable positive.

Result 2: the slope is substantial positive $(x, y) \in \Re_+^{*2}$ as $y_0 \in \left(\frac{n}{m} A_+\right)^{\frac{1}{2n+1}} x_0^{\frac{2m+1}{2n+1}}$

Situation where the slope is infinitely big positive:

$\frac{m}{n}\frac{y_0^{2n+1}}{x_0^{2m+1}} \in G_+ \text{ is equivalent } y_0 \in \left(\frac{n}{m} G_+\right)^{\frac{1}{2n+1}} x_0^{\frac{2m+1}{2n+1}}$ if the slope is infinitely great then :

Result 3: the slope is substantial positive as if $(x, y) \in \Re_+^{*2}$ as $y_0 \in \left(\frac{n}{m} G_+\right)^{\frac{1}{2n+1}} x_0^{\frac{2m+1}{2n+1}}$

Parameters monitoring the shape of the curves (general case): In the general case V_k and h_k are equal:

$$V_k = \left(1 - \left(\frac{k-1}{k}\right)^{-\frac{1}{2n}}\right)\left(\frac{ak}{k-1}\right)^{-\frac{1}{2n}}$$

k>1 Vertical thickness.

$$h_k = \left(1 - k^{-\frac{1}{2m}}\right)\left(\frac{a}{k}\right)^{-\frac{1}{2m}}$$

k>1 horizontal thickness bands

The curve $C_k(m,n)$ correspond to $E(m,n,a)$.

CONCLUSION

In this study we have introduced a non-standard analysis technique and regionalization for resolving algebraic curves formalized by algebraic equations

REFERENCES

1. Robinson, A., 1966. Non-Standard analysis. North-Holland.
2. Nelson, E. 1977. Interior set theory, Bull Am. Math. Soc. 83: 1165- 1198.
3. Bobo Seke, 1988. Méthodes de régionalisations, quelques applications I.R.M.A, Strasbourg.