**Journal of Mathematics and Statistics 9 (1): 24-28, 2013**
ISSN 1549-3644

doi:10.3844/jmssp.2013.24.28 Published Online 9 (1) 2013 (http://www.thescipub.com/jmss.toc)

# A MODIFIED METHOD FOR SOLVING SYSTEM OF NONLINEAR EQUATIONS

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Received 2013-01-28, Revised 2013-02-16; Accepted 2013-03-15

## ABSTRACT

Solving systems of nonlinear equation is a great important which arises in various branches of science and engineering. In the last decades, several numerical techniques were proposed to solve these problems. In this study, we propose a modified of iterative method which is based on the idea of Newton method and Fixed point iteration method. The proposed method has been illustrated with several examples from the reference. The numerical results indicate that this proposed method provide the good performance of iterations.

**Keywords:** System of Nonlinear Equation, Newton Method, Fixed Point Iteration

## 1. INTRODUCTION

Solving systems of nonlinear equations is a great importance, because these systems frequently arise in various branches of pure and applied sciences.

The general form of a system of nonlinear equations is Equation 1:

$$f_1(x_1, x_2, \ldots, x_n) = 0, f_2(x_1, x_2, \ldots, x_n) = 0, f_n(x_1, x_2, \ldots, x_n) = 0 \quad (1)$$

where, each function $f_i$ can be thought of as mapping a vector $x = (x_1, x_2, \ldots, x_n)$ of the n–dimensional space $R^n$, into the real line R. The system can alternatively be represented by defining a functional F, mapping $R^n$ into $R^n$ by.

$F(x_1, x_2, \ldots, x_n) = (f_1(x_1, x_2, \ldots, x_n), \ldots, f_n(x_1, x_2, \ldots, x_n))^T$ Using vector notation to represent the variables $x_1, x_2, \ldots, x_n$, a system (1) can be written as the form:

$$F(x) = 0$$

The functions $f_1, f_2, \ldots, f_n$ are called the coordinate functions of F (Burden and Farires, 2010).

Recently, several iterative methods have been used to solve nonlinear equations and the system of nonlinear equations (Awawdeh, 2009; Noor, 2010; Cordero *et al*., 2011; Sharma and Sharma, 2011; Vahidi *et al*., 2012). Wang (2011) using a third order family of Newton-Like iteration method for solving nonlinear equations; Ozel (2010) has considered a new decomposition method for solving the system of nonlinear equations. Saha (2010) has presented a modified method to solving nonlinear equations by hybridising the results of Newton method and fixed point iteration method. Kim *et al*. (2010) developed a new scheme for the construction of iterative methods for the solution of nonlinear equations and giving a new class of methods from any iterative method. Furthermore, several iterative methods have been developed for solving the system of nonlinear equations by using various techniques such as Newton's method, Revised Adomian decomposition method, homotopy perturbation method, Householder iterative method (Darvishi, 2009; Noor and Waseem, 2009; Hosseini and Kafash, 2010; Darvishi and Shin, 2011; Hafiz and Bahgat, 2012a; 2012b; Noor *et al*., 2012).

It is the purpose of this study to introduce a new improvement of Newton method by fixed point iteration method. We extend the Saha (2010) method to solve systems of nonlinear equations. Some examples are tested and the obtained results suggest that this newly improvement technique introduces a promising tool and powerful improvement for solving a System of Nonlinear Equations.

### 1.1. Description of an Iterative Method

Consider a nonlinear equation:

$$f(x) = 0 \tag{2}$$

We assume that the Equation (2) admits a unique solution $x^*$.

In Newton method the all known iterative formula used to find the real root is Equation 3:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \tag{3}$$

In fixed point iteration method (2) will be rewritten in the form:

$$x = g(x) \tag{4}$$

Equation 4 which is equivalent to (2) will converge to a real root in the interval D if $|g`(x)|<1$ for all x in D provided the initial approximation $x_0$ is chosen in D.

Choose the initial approximation $x_0$ then $(x_0, g(x_0))$ is a point on the curve Equation 5:

$$y = g(x) \tag{5}$$

The equation of the tangent to the curve given by (5) at the point $(x_0, g(x_0))$ is Equation 6:

$$y - g(x_0) = g'(x_0)(x - x_0) \tag{6}$$

Now we consider the line Equation 7:

$$y = x \tag{7}$$

Substituting y = x in (6) we have:

$$x - g(x_0) = g'(x_0)(x - x_0)$$
$$x[1 - g'(x_0)] = g(x_0) - g'(x_0)x_0 x = \frac{g(x_0) - g'(x_0)x_0}{1 - g'(x_0)}$$

which produces the following iteration scheme Equations 8:

$$x_{k+1} = \frac{g(x_k) - g'(x_k)x_k}{1 - g'(x_k)} \tag{8}$$

### 1.2. The N-Dimensional Case

The Newton method (Gautschi, 2011; Sauer, 2011) is commonly used for solving such systems Equation 9:

$$F(x) = 0 \tag{9}$$

where, $F: \Omega \subseteq R^n \to R^n$ is defined Equation 10:

$$x_{k+1} = x_k - \frac{F(x_k)}{F'(x_k)} \tag{10}$$

where, $F'(x_k)$ is the Jacobian matrix in point $x_k$.

In fixed point iteration method (9) will be rewritten in the form x = g(x) We rewrite Equation 8 to solve the nonlinear system F(x) = 0, this produces the following iteration scheme Equations 11:

$$x_{k+1} = [I - g'(x_k)]^{-1}[g(x_k) - g'(x_k)x_k] \tag{11}$$

where, I is an identity matrix.

### 1.3. Numerical Examples

We present some examples to illustrate the efficiency of our proposed methods, we solve four systems of nonlinear equations and one of a nonlinear boundary value problem. The following tables show the Number of Iterations (NI) to receive the required solution. For all test problems the stop criteria is $\| F(x) \| < 10^{-9}$.

### Example 1

Consider the following system of nonlinear equations:

$$\begin{aligned} x_1^2 + x_2^2 - 2 &= 0 \\ x_1^2 - x_2 &= 0 \end{aligned}$$

The exact solutions are $x^* = (x_1^*, x_2^*)^T = (1,1)^T$. To solve this system, we set $x_0 = (0.01,0)^T$ as an initial value. The results are presented in **Table 1**.

### Example 2

Consider the following system of nonlinear equations (Hosseini and Kafash, 2010):

$$\begin{aligned} x_1^3 + x_2^3 - 6x_1 + 3 &= 0 \\ x_1^3 - x_2^3 - 6x_2 - 2 &= 0 \end{aligned}$$

The exact solutions are $x^* = (x_1^*, x_2^*)^T = (1,1)^T =$ $(0.532370372327903,\ 0.351257447590883)^T$ To solve this system, we set $x_0 = (0.53,0.35)$ as an initial value. The results are presented in **Table 2**.

**Table 1.** Numerical results for Example 1

| | Newton method | | Present method | |
|---|---|---|---|---|
| NI | $X_1$ | $X_2$ | $X_1$ | $X_2$ |
| 1 | 100.005000000 | 2.000000000 | 1.414213562 | 0.028184271 |
| 2 | 50.008499700 | 1.200000000 | 1.376740706 | 1.894010756 |
| 3 | 25.014365777 | 1.011764706 | 1.012764344 | 0.893212825 |
| 4 | 12.527172318 | 1.000045777 | 0.874295814 | 0.821483562 |
| 5 | 6.303499396 | 1.000000001 | 1.009689551 | 1.003473386 |
| 6 | 3.231070718 | 1.000000000 | 0.999759947 | 0.999740042 |
| 7 | 1.770282823 | 1.000000000 | 1.000000032 | 1.000000035 |
| 8 | 1.167582157 | 1.000000000 | 1.000000000 | 1.000000000 |
| 9 | 1.012026468 | 1.000000000 | | |
| 10 | 1.000071459 | 1.000000000 | | |
| 11 | 1.000000003 | 1.000000000 | | |
| 12 | 1.000000000 | 1.000000000 | | |

**Table 2.** Numerical results for Example 2

| | Newton method | | Present method | |
|---|---|---|---|---|
| NI | $X_1$ | $X_2$ | $X_1$ | $X_2$ |
| 1 | 0.487178345 | -0.282914615 | 0.487178345 | -0.282914615 |
| 2 | 0.518276386 | -0.305686078 | 0.518276386 | -0.305686078 |
| 3 | 0.518485025 | -0.305357504 | 0.518485025 | -0.305357504 |
| 4 | 0.518485020 | -0.305357478 | 0.518485020 | -0.305357478 |

**Table 3.** Numerical results for Example 3

| | Newton method | | | Present method | | |
|---|---|---|---|---|---|---|
| NI | $X_1$ | $X_2$ | $X_3$ | $X_1$ | $X_2$ | $X_3$ |
| 1 | 4.708415325 | 1.987763973 | -0.473598777 | 5.651617720 | 0.557742445 | -0.473598777 |
| 2 | 0.502420021 | 0.894108971 | -0.473661290 | 0.511999092 | -0.032012717 | -0.488991715 |
| 3 | 0.497590942 | 0.402121042 | -0.513529635 | 0.500035088 | 0.000010168 | -0.523572254 |
| 4 | 0.501217278 | 0.161077952 | -0.519381380 | 0.500000000 | -0.000000000 | -0.523598776 |
| 5 | 0.500431980 | 0.049727088 | -0.522300621 | | | |
| 6 | 0.500074560 | 0.008268410 | -0.523382611 | | | |
| 7 | 0.500002880 | 0.000316299 | -0.523590503 | | | |
| 8 | 0.500000005 | 0.000000500 | -0.523598763 | | | |
| 9 | 0.500000000 | 0.000000000 | -0.523598776 | | | |

**Table 4.** Numerical results for Example 4

| | Number of iterations | | |
|---|---|---|---|
| Method | m = 50 | m = 75 | m = 100 |
| Newton method | 6 | 6 | 6 |
| Present method | 4 | 4 | 4 |

**Table 5.** Numerical results for Example 5

| | Number of iterations | | |
|---|---|---|---|
| Method | M = 50 | M = 75 | M = 100 |
| Newton method | 6 | 7 | 7 |
| Present method | 6 | 7 | 7 |

## Example 3

Consider the following system of nonlinear equations (Awawdeh, 2009):

$$\begin{aligned} 3x_1 - \cos(x_2x_3) - 0.5 &= 0 \\ x_1^2 - 81(x_2+0.1)^2 + \sin x_3 + 1.06 &= 0 \\ e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} &= 0 \end{aligned}$$

The exact solutions are $x^* = (x_1^*, x_2^*, x_3^*)^T = (0.5, 0, -0.5235987755982)^T$. To solve this system, we set $x_0 = (5, 4, 2)^T$ as an initial value. The results are presented in **Table 3**.

### Example 4

Consider the following system of nonlinear equations (Darvishi and Shin, 2011):

$$x_i^2 - \cos(x_i - 1) = 0, \quad i = 1,2,\ldots,m$$

The exact solutions are $x^* = (x_1^*, x_2^*, \ldots, x_m^*)^T = (1,1,\ldots,1)^T$. To solve this system, we set $x_0 = (0.5, 0.5,\ldots, 0.5)^T$ as an initial value. The results are presented in **Table 4**.

### Example 5

Consider the nonlinear boundary value problem (Noor and Waseem, 2009):

$$y'' = -(y')^2 - y + \ln x, \quad 1£x£2, \; y(1) = 0, \; y(2) = \ln 2$$

Whose exact solutions is y = Inx. We consider the following partition of the interval:

$$x_0 = 1, \; x_n = 2, \; x_j = x_0 + jh, \; h = \frac{1}{m}, \; j = 1,2,\ldots,m-1$$

Let us define now:

$$y_0 = y(x_0) = 0, \; y_m = \ln 2, \; y_i = f(x_i), \; i = 1,2,\ldots,m-1$$

If we discretize the problem by using the second order finite differences method defined by the numerical formulas:

$$y_i' = \frac{y_{i+1} - y_{i-1}}{2h}, \; i = 1,2,\ldots,m-1, y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}, \; i = 1,2,\ldots,m-1$$

Then, we obtain a (m-1) × (m-1) system of nonlinear equations:

$$4y_2 + y_2^2 + 4y_1(h^2 - 2) - 4h^2\ln x_1 = 0, 4(y_{i+1} + y_{i-1}) + (y_{i+1} - y_{i-1})^2 + 4y_i(h^2 - 2) - 4h^2\ln x_i = 0, i = 2,\ldots,m-2,$$
$$4(\text{In}2 + y_{m-2}) + (\text{In}2 - y_{m-2}) + 4y_{m-1}(h^2 - 2) - 4h^2\text{In}x_{m-1} = 0$$

we take $X_0$ with $y_k^{(0)} = \ln\left(\frac{k}{10}\right), \; k = 1,2,\ldots,m-1$, as a starting point. The results are presented in **Table 5**.

## 2.CONCLUSION

In this study, we have demonstrated the applicability of the modified method for the system of nonlinear equations with the help of some concrete examples. The results show that: the proposed problem can be solved by the proposed method.

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