Journal of Mathematics and Statistics, 9 (2): 77-83, 2013
ISSN 1549-3644

doi:10.3844/jmssp.2013.77.83 Published Online 9 (2) 2013 (http://www.thescipub.com/jmss.toc)

OPTIMIZING AN ALUMINUM EXTRUSION PROCESS

Mohammed Ali Hajeeh

Techno-Economics Division, Kuwait Institute for Scientific Research, P.O. Box 24885, Safat-13109, Kuwait

Received 2012-09-12, Revised 2013-02-28; Accepted 2013-04-23

ABSTRACT

Minimizing the amount of scrap generated in an aluminum extrusion process. An optimizing model is constructed in order to select the best cutting patterns of aluminum logs and billets of various sizes and shapes. The model applied to real data obtained from an existing extrusion factory in Kuwait. Results from using the suggested model provided substantial reductions in the amount of scrap generated. Using sound mathematical approaches contribute significantly in reducing waste and savings when compared to the existing non scientific techniques.

Keywords: Minimizing, Cutting Patterns, Aluminum Logs, Mathematical Approaches

1. INTRODUCTION

Aluminum is the third most abundant element in the Earth’s crust and it constitutes 7.3 percent by mass. The Aluminum industry contributes significantly to the global economy as well as too many individual economies. The industry employs over a million people worldwide. Aluminum smelting is a capital-intensive, technology-driven industry concentrated in a few relatively dominant companies. Aluminum consumption has enjoyed substantial average growth over the last few decades due to general economic growth and to its substitution of other materials. Kuwait is a member of the Gulf Cooperation Council (GCC); the GCC countries will boost their share of global aluminum output to 15-17% by the end of the decade. There are two types of aluminum industries; the first is the extrusion industry where profiles of different sizes, colors and shapes are produced, while the second is the fabrication industry where various products such as windows, fences and doors are designed from aluminum profiles.

In the aluminum extrusion industry, logs and billets are cut using various stock cutting patterns; the amount of scrap generated is dependent on the cutting method used. The Stock Cutting Problem (SCP) is discussed thoroughly in the literature. One of the first articles was presented by Gilmore and Gomory (1961) where integer programming was utilized for the cutting stock problem. The problem compromised a large number of variables which generally makes the computation infeasible. Gilmore and Gomory (1964) examined the cutting stock problems involving two or more dimensions. Haessler (1971) described a heuristic procedure for scheduling production-rolls of paper through a finishing operation to cut them down to finished roll sizes. The objective was to minimize the cost of trim-loss and that of the reprocessing. Covesrdale and Wharton (1976) presented a heuristic procedure for a nonlinear cutting stock problem; the problem was solved using the pattern enumeration technique.

Sumichrast (1986) addressed a scheduling problem in the woven fiber glass industry as an example of the cutting stock problem with the objective of controlling the wasted production capacity rather than wasted material. A heuristic was developed for the purpose of scheduling the production process. Stadtler (1990) used the column generation method of Gilmore and Gomory (1964) for minimizing the amount of scrap generated from fabricated aluminum made for window frames. Krichagina *et al*. (1998) examined the cutting process of sheets in a paper plant. The main objective was to minimize the long-run average cost of paper waste. In this regard, a two step procedure consisting of linear programming and Brownian control was developed.

Liang *et al*. (2002) applied an evolutionary algorithm (EP) for cutting stock problems with and without contiguity. Results showed that the EP algorithm is more effective and superior when compared to the genetic algorithm used. Parada *et al*. (2003) proposed a meta-heuristic approach for solving a non-guillotine stock cutting problem. The approach was a combination of the principles of the constructive and evolutive methods. The results showed an error reduction of around 2%. Hifi (2004) proposed an algorithm for solving a two dimensional constrained cutting stock problem. In the algorithm and for depth search, hybrid approach combining hill climbing strategies and dynamic programming were employed. Cui (2005) developed an algorithm that utilizes the knapsack algorithm and an implicit enumeration technique. The algorithm was applied to real cutting stock data of the manufacture electric generators. Khalifa *et al*. (2006) built a one dimensional cutting stock problem using genetic algorithm. The objective was to reduce the amount of waste generated in constructing steel bars.

Saad *et al*. (2007) addressed the problem of scrap generated from cutting cylindrical logs produced by an aluminum extrusion company. In this regard, a multi-objective cutting stock problem was constructed. A solution procedure was developed considering the scrap generated as a fuzzy parameter. Chen (2008) presented a recursive heuristic algorithm for the constrained two-dimensional stock cutting problems. The algorithm was tested and the computational results produced good solutions in short computing time for problems of different scales. Alves *et al*. (2009) used several constrained and non constrained integer programming using column generation. lower bounds for the different minimzation patterns were derived. Using actual data,the outcome of these models showed improvement of the lower bounds. Hajeeh (2010) addressed the problem of waste generated in an aluminum fabrication industry. A heuristic was propped for optimizing the cutting of aluminum profiles. The heuristic produced less scrap when compared to the existing procedure used in the company. Macedo *et al*. (2010) proposed an integer linear programming model to solve the two-dimensional stock cutting problem with guillotine constraint. A computer software was used to examine the behavior of the models with data from a wood industry. The lower bound of the model was found to be superior to those of other methods.

Kasimbeyli *et al*. (2011) proposed a linear integer programing with two contarcting objectives for a one-dimensional cutting problem. A special heuristic algorithm was used to find the optimim cutting pattern. Berberler *et al*. (2011) developed a dynamic programming algorithm to address the one-dimensional stock cutting problem. The results obtained from this algorithm was compared to others and results showed its efficeincy and superiority. Cui and Huang (2012) proposed a heuristic to address constrained T-shaped patterns with the objective of maximizing the pattern value and meeting demand. The computation of 58 benchmark instances showed that the algorithm is superior to the two-stage patterns approaches. De Valle *et al*. (2012) developed an algorithm based on non-fit polygen to examine the two dimensional cutting/packing problem. The algorithm aslo solved problems with items of irregular shapes. Mobasher and Ekici (2013) developed a mixed integer linear and used the column generation method to the study a cutting stock problem with set up cost. The main objective was to find a cutting pattren at minimum production cost.

In the current research work, the extrusion process in a specific industry in Kuwait is thoroughly studied with objective of finding ways for reducing the large amount of scrap generated. The article organized as follows: it start by describing the aluminum profile production process, the amount of scrap generated in the chosen industry from using the existing cutting techniques. Next, the structure of the developed optimization models is provided. For illustration, an example is presented to compare the amount of scarp generated using the existing conventional cutting patterns and the size of scrap generate from the proposed optimization model. Results and discussion section comes next, the article ends with concluding remarks.

2. MATERIALS AND METHODS

2.1. Aluminum Profile Production

Aluminum profile production (extrusion) process passes through several stages starting with castings where logs are produced; the logs are next cut into standard billets and are put into extrusion machine to manufacture profiles of different shapes and sizes. The extruded aluminum profiles are placed in the aging furnace in order to increase their durability and strength. Next, the profiles are polished thoroughly and depending on request are either sent for paining, or anodizing before shipping to the customer. In **Fig. 1**, the detailed process is presented for a specific extrusion company in Kuwait.

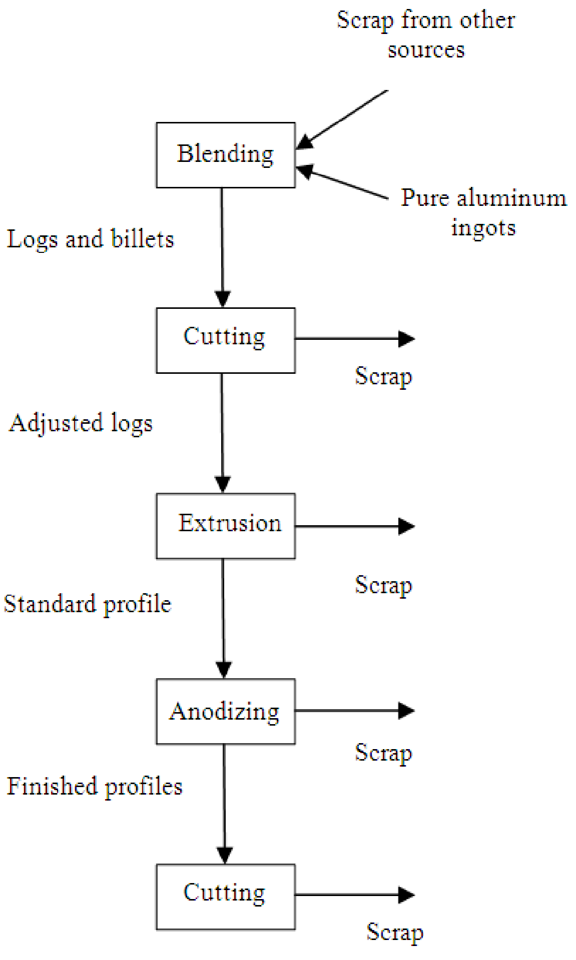

Fig. 1. Aluminum profile production process

The type of billets and logs used in the extrusion process in the company along with their lengths and weight is shown in **Table 1**. The monthly weight and percentage of scrap generated during a specific year in the aluminum extrusion by the same company is as given in **Table 2**.

2.2. Optimization Models

An efficient model for minimizing scrap in the profile production process is based on developing cutting patterns, where each cutting pattern uses several billets and the total weight cut is less than the total weight of the log used. All possible patterns are investigated and the weight of the scrap generated is calculated. Two models are presented in this research work, details are given below.

Table 1. Lengths and weights of standard billets and logs used by the aluminum extrusion company

Billet Type	Length (m)	Weight (kg)	Log Type	Length (m)	Weight (kg)
1	0.48	32.16	1	2.59	173.4
2	0.52	34.25	2	2.64	176.8
3	0.56	37.55	3	2.69	180.2
4	0.58	38.78	4	2.74	183.6
5	0.61	40.55	5	2.79	187.0
6	0.65	43.55	6	2.84	190.4

Table 2. Weight and percentage of monthly total production scrapped in the extrusion stage

Month	Total production (kg)	Scrap weight (kg)	Percentage scrap (%)
January	249255	64722	26
February	280077	75978	27
March	295545	77954	27
April	343386	91102	27
May	245831	65408	27
June	321935	68588	21
July	268012	61195	23
August	219373	52435	24
September	317112	78005	25
October	278610	63709	23
November	347567	83844	24
December	385956	88688	23
Mean	296055	72636	25
Standard deviation	48643	11328	

Model I

Two models are presented in this research work. The first is shown in (1) which is based on different billets:

Minimize $Z = W_1 + W_2 + W_3$

Subject to Equation 1:

$$\begin{aligned}
&\nu_1\xi_1 + \nu_2\xi_1 + \nu_3\xi_3 + \nu_4\xi_4 + \nu_5\xi_5 + \nu_6\xi_1 = W_1 \\
&\omega_1\xi_1 + \omega_2\xi_2 + \omega_3\xi_3 + \omega_4\xi_4 + \omega_5\xi_5 + \omega_6\xi_6 - W_L y = W_2 \\
&\omega_1\xi_1 + \omega_2\xi_2 + \omega_3\xi_3 + \omega_4\xi_4 + \omega_5\xi_5 + \omega_6\xi_6 - W_F y = W_3 \\
&y, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6 \geq 0 \text{ and integers}
\end{aligned} \tag{1}$$

Where:

Z = Total weight of the scarp in kg
W_1 = Weight of scrap (kg) generated per 1.5 inch log (0.0381 m) of each billet used to extrude the desired the desired profile
W_2 = Weight of scrap (kg) generated from log cutting
W_3 = Weight of scrap (kg) generated from producing longer profiles than demanded
ν_j = Weight (kg) of producing type j billet, j = 1,…,6
ω_j = Weight (kg) of log used to produced the j^{th} billet, j = 1,…,6
ξ_j = Number of type j billet used
y = Number of logs used
W_L = Weight (kg) of each log
W_F = Total weight (kg) of profiles demanded

Model II

Although the above problem provides a good solution to the problem, however it has one disadvantage in that, where more than on log is needed, the model considers all logs to be one long log. A superior and more efficient model is based on cutting patterns as given in (2). Equation (3) represents the nenegtivity constraint.

$$\text{Minimize } Z = \sum_{j=1}^{J} S_j \rho_j$$

Subject to Equation 2:

$$\sum_{j=1}^{6} Q_j \rho_j \geq Q_{ML} \quad (2)$$

$$\rho_j \text{ non negative integers} \quad (3)$$

Where:

ρ_j = Number of cutting pattern j, j = 1, …,6
S_j = Amount of scrap generated from cutting pattern j, j = 1, …,6
Q_j = Weight of cutting pattern j (kg), j = 1, …,6
Q_{ML} = Total weight of profiles demanded

The different cutting billet patterns of 1.5 m log used in the mathematical model are given in **Table 3** along with the total length of the different billet combinations (patterns) and the amount of scrap generated in meters.

2.3. Example

As an example, the proposed model II has been used on die number 158B/ 7 for a demand 441 kg. Detailed mathematical programming formulation is as follows noting that Z and Q represent the amount of scrap generated from using two logs:

Minimize (Z + Q):

Subject to:

$7.04X_1 +5.04 X_2 +34.04 X_3 +33.04X_4 +31.04 X_5 +28.04 X_6 +35.07 X_7 +32.04X_8 +31.04 X_9 +29.04 X_{10} +26.04X_{11} +29.04 X_{12} +28.04 X_{13} +26.04 X_{14} +23.04 X_{15} +27.04 X_{16} + 25.04X_{17} +22.04 X_{18} +23.04 X_{19} +20.04 X_{20} +17.04 X_{21} - Z = 0$

$96 X_1 +98 X_2 +69 X_3 +604X_4 +72 X_5 + 72 X_6 +68X_7 +71X_8 + 72X_9 +74 X_{10} +77X_{11} +74 X_{12} +75 X_{13} +77 X_{14} + 80 X_{15} +76 X_{16} +784X_{17} +81 X_{18} +80 X_{19} +83 X_{20} + 86 X_{21} - Q = 0$

END
GIN X_1
GIN X_2
GIN X_3
GIN X_4
GIN X_5
GIN X_6
GIN X_7
GIN X_8
GIN X_9
GIN X_{10}
GIN X_{11}
GIN X_{12}
GIN X_{13}
GIN X_{14}
GIN X_{15}
GIN X_{16}
GIN X_{17}
GIN X_{18}
GIN X_{19}
GIN X_{20}
GIN X_{21}

Objective Function Value: 74.2.

Table 3. Scrap generated from different cutting patterns (γ_j) of 1.5 m Log (100.5 kg)

Cutting patterns (γ_j)	Standard billets 0.48	0.52	0.56	0.58	0.61	0.65	Total length (m)	Total scrap (m)
γ_1	3						1.44	0.174
γ_2	2	1					1.48	0.134
γ_3	1		1				1.04	0.536
γ_4	1			1			1.06	0.516
γ_5	1				1		1.09	0.486
γ_6	1					1	1.13	0.446
γ_7	1	2					1.04	0.536
γ_8		1	1				1.08	0.486
γ_9		1		1			1.10	0.476
γ_{10}		1			1		1.13	0.446
γ_{11}		1				1	1.17	0.406
γ_{12}			2				1.12	0.456
γ_{13}			1	1			1.14	0.436
γ_{14}			1		1		1.17	0.406
γ_{15}			1			1	1.21	0.366
γ_{16}				2			1.16	0.416
γ_{17}				1	1		1.19	0.346
γ_{18}				1		1	1.23	0.356
γ_{19}					2		1.22	0.316
γ_{20}					1	1	1.26	0.134
γ_{21}						2	1.30	0.276

Total Scrap (kg.) = 100-total weight +2.5(number of billets used)

Table 4. Comparison of extrusion stage scrap generated by AEC's conventional method and the proposed mathematical model

No	Die number	Weight demanded (kg)	Total scrap generated (cm) Conventional method	Optimization model
1	158 B/7	441.00	275.350	74.215
2	861 A/3	450.00	123.400	65.200
3	861 A/3	675.00	204.350	42.280
4	113 D/12	396.00	226.960	119.120
5	7613	153.00	63.720	52.400
6	2211	165.00	65.510	41.080
7	1096	259.00	185.430	49.920
8	1037A/6	2574.00	311.360	208.080
9	863A/3	888.00	76.830	64.320
10	126A/B	244.80	328.120	205.900
11	463	130.05	196.400	146.560
	Total Scrap	2034.99	1145.105	

3. RESULTS

Table 3 provides the amount of scrap generated by different cutting patterns**.** The amount of scarp generated is less than one meter. As shown pattern 3 produces the 0.536 meters, while patterns 2 and 20 produce around 0.134 meters which is the least. **Table 4** presents the amount of scrap generated resulted for ten Dies along with the total weight demand. The amount of scrap generated using the conventional method used in the company is compared to that using the optimization model. The maximum scrap is genertaed by Die 1037A/6 with an amount of 208.08 kg while the least is produce by 2211 with around 41 kg.

4. DISCUSSION

When comparing the total amount of scrap generated for the different Dies as shown in **Table 4**, it is found that it is around 56% on average. The total scrap generated produced using the conventional procedure is around 2035 cm whereas it is around 1145 using the suggested optimization model.

5. CONCLUSION

The aluminum extrusion process produces a sizable amount of scarp; this mainly attributed to the techniques used and the lack of modern scientific experience of the staff. In order to reduce the large amount of scrap, a thorough evaluation of the exiting cutting method should be carrying out. In addition to reduce scrap, an effective an optimal cutting method will contribute to efficient uses of time and other resources. For example the use of the mathematical developed, the reduction of the amount of scrap generated ranged from 21-82% and this constitutes large saving.

Efficient scientific approaches and tools should be used in the different processes within industries; optimization techniques are one of the strong tools that produce good results. It is recommended that the extrusion company should substitute the existing methods with more cost effective approaches which are based on sound scientific ones.

6. REFERENCES

Alves, C., R. Macedo and J.V.D. Carvalho, 2009. New lower bounds based on column generation and constraint programming for the pattern minimization problem. Comput. Operat. Res., 36: 2944-2954. DOI: 10.1016/j.cor.2009.01.008

Berberler, M.E., U. Nuriyev and A. Yildirim, 2011. A software for the one-dimensional cutting stock problem. J. King Saud Univ., 23910: 69-76. DOI: 10.1016/j.jksus.2010.06.009

Chen, Y., 2008. A recursive algorithm for constrained two-dimensional cutting problems. Comput. Optim. Appli., 4193: 337-348. DOI: 10.1007/s10589-007-9103-3

Covesrdale, L. and F. Wharton, 1976. An improved heuristic procedure for a nonlinear cutting stock problem. Manage. Sci., 23: 79-86. DOI: 10.1287/mnsc.23.1.78

Cui, Y. and B. Huang, 2012. Heuristic for constrained T-shape cutting patterns of rectangular pieces. Comput. Operat. Res., 39: 3031-3039. DOI: 10.1016/j.cor.2012.03.001

Cui, Y., 2005. A cutting stock problem and its solution in the manufacturing industry of large electric generators. Comput. Operat. Res., 32: 1709-1721. DOI: 10.1016/j.cor.2003.11.022

De Valle, A.M., T.A.D. Queiroz, F.K. Miyazawa and E.C. Xavier, 2012. Heuristics for two-dimensional knapsack and cutting stock problems with items of irregular shape. Exp. Syst. Appli., 39: 12589-12598. DOI: 10.1016/j.eswa.2012.05.025

Gilmore, P.C. and R.E. Gomory, 1961. A linear programming approach to the cutting-stock problem. Operat. Res., 1: 849-859. DOI: 10.1287/opre.9.6.849

Gilmore, P.C. and R.E. Gomory, 1964. Multistage cutting stock problems of two and more dimensions. Operat. Res., 13: 94-120. DOI: 10.1287/opre.13.1.94

Haessler, R.W., 1971. A heuristic programming solution to a nonlinear cutting stock problem. Manage. Sci., 17: B793-B802. DOI: 10.1287/mnsc.17.12.B793

Hajeeh, M.A., 2010. Mathematical model for waste reduction in aluminum fabrication industry in Kuwait. J. Math. Stat., 6: 110-115. DOI: 10.3844/jmssp.2010.110.115

Hifi, M., 2004. Dynamic programming and hill-climbing techniques for constrained two-dimensional cutting stock problems. J. Combinatorial Optim., 8: 65-84. DOI: 10.1023/B:JOCO.0000021938.49750.91

Kasimbeyli, N., T. Sarac and R. Kasimbeyli, 2011. A two-objective mathematical model without cutting patterns for one-dimensional assortment problems. J. Combinatorial Optim. Appl. Math., 235: 4663-4674. DOI: 10.1016/j.cam.2010.07.019

Khalifa, Y., O. Salem and A. Shahin, 2006. Cutting stock waste reduction using genetic algorithms. Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation, Jul. 08-12, ACM New York, USA. DOI: 10.1145/1143997.1144272

Krichagina, E.V., R. Rubio, M.I. Taskar and L.M. Wein, 1998. A dynamic stochastic stock-cutting problem. Operat. Res., 46: 690-701. DOI: 10.1287/opre.46.5.690

Liang, K.H., X. Yao, C. Newton and D. Hoffman, 2002. A new evolutionary approach to cutting stock problems with and without contiguity. Comput. Operat. Res., 29: 1641-1659. DOI: 10.1016/S0305-0548(01)00039-9

Macedo, R., C. Alves and J.M.V.D. Carvalho, 2010. Arc-flow model for the two-dimensional guillotine cutting stock problem. Comput. Operat. Res., 37: 991-1001. DOI: 10.1016/j.cor.2009.08.005

Mobasher, A. and A. Ekici, 2013. Solution approaches for the cutting stock problem with setup cost. Comput. Operat. Res., 40: 225-235. DOI: 10.1016/j.cor.2012.06.007

Parada, V., L. Pradenas, M. Solar and R. Palma, 2003. A hybrid algorithm for the non-guillotine cutting problem. Annals Operat. Res., 117: 151-163. DOI: 10.1023/A:1021521523118

Saad, O.M., M.K. El-shafei and L.E. Ezzat, 2007. On treating multiobjective cutting stock problem in the aluminum industry under fuzzy environment. J. Applied Sci. Res., 3: 938-945. DOI: 10.3923/tasr.2007.374.385

Stadtler, H., 1990. A one-dimensional cutting stock problem in the aluminium industry and its solution. Eur. J. Operat. Res., 44: 209-223. DOI: 10.1016/0377-2217(90)90356-G

Sumichrast, R.T., 1986. A new cutting-stock heuristic for scheduling production. Comput. Operat. Res., 13: 713-719. DOI: 10.1016/0305-0548(86)90027-4