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FUZZY MODEL OPTIMIZATION FOR TIME SERIES DATA USING A TRANSLATION IN THE EXTENT OF MEAN ERROR

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ABSTRACT

Recently, many researchers in the field of writing about the prediction of stock price forecasting, electricity load demand and academic enrollment using fuzzy methods. However, in general, modeling does not consider the model position to actual data yet where it means that error is not been handled optimally. The error that is not managed well can reduce the accuracy of the forecasting. Therefore, the paper will discuss reducing error using model translation. The error that will be reduced is Mean Square Error (MSE). Here, the analysis is done mathematically and the empirical study is done by applying translation to fuzzy model for enrollment forecasting at the Alabama University. The results of this analysis show that the translation in the extent of mean error can reduce the MSE.

Keywords: Fuzzy Model, Time Series, Model Translation, Forecasting

1. INTRODUCTION

Everyday, the individual investors, the managers of stock funds and the financial analyst try to predict the activities of stock price movements based on their professional knowledge or use a tool they have to analyze the stock price. High accuracy is their priority, because an accurate prediction gives more advantages. Therefore, they keep finding ways to predict stock price more accurately.

Some researchers have improved time series modeling using fuzzy method. Fuzzy method is chosen in modeling based on a consideration that fuzzy can accept the input of linguistic variable as experienced by the economist's (Wang, 1997).

The use of fuzzy method in time series forecasting is becoming increasingly widespread. Song and Chissom (1993a; 1993b; 1994) have studied fuzzy application to predict university enrollment. Time series using fuzzy model has been applied to predict peak load electricity demand (Ismail *et al*., 2009) and has been applied to predict stock price (Egrioglu, 2014; Kao *et al*., 2013; Nurhayadi *et al*., 2014; Singh and Borah, 2014). Rodger (2014) has used fuzzy model to predict the need of natural gas and the energy cost savings in public buildings. While Birek *et al*. (2014) has applied fuzzy time series for water leakage forecasting in a water supply company.

In addition to the use of fuzzy model that becomes more and more widespread, the researchers have also been trying to improve the quality of forecasting in many ways. Cheng *et al*. (2006) has generated fuzzy time series model by grouping fuzzy relations based on their antecedents and it gives better results. Giving weights based on the numbers of group members can improve the model accuracy (Yu, 2004; Suhartono and Lee, 2011; Nurhayadi *et al*., 2012).

Singh and Borah (2013) and Wang *et al*. (2013) have improved the quality of model using various lengths of intervals. After that, Singh and Borah (2014) have tried to increase the quality using particle swarm optimization.

There are also some researchers who have combined fuzzy method and another method. Allahverdi *et al*. (2011), Wei *et al*. (2014) and Sethukkarasi and Kannan (2012) combined fuzzy method and neural network to improve the quality of model. Rahoma *et al*. (2011)

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using neuro fuzzy for the prediction of daily solar radiation. Wavelet has also been used as a preprocessing of fuzzy modeling (Popoola, 2007; Zhu *et al*., 2014) for the quality improvement.

Lee *et al*. (2012) said that recent studies show that the newer and more advanced forecasting techniques tend to result in improved forecast accuracy, but no clear evidence shows that any one model can consistently outperform other models in the forecasting competition. The quality comparison between two models is generally difficult to be done, because they usually have different basics in their modeling.

In this study, we increase the optimization by modifying the generated model. Generally, the modeling using fuzzy method does not consider the model precision to the sample data yet. Generated estimation model can be relatively higher or lower than the real data. Therefore, the resulted model needs to be optimized. The optimization can be done by sliding the model up or down.

Agrawal *et al*. (1995), Argyros and Charis (2003) and Kontaki *et al*. (2005) have ever used transformation to find the similarity of two sub time series. Graphically, it is done by squeezing two sub time series that have been normalized and then the similarity can be seen from the closeness of each pair of points in two sub time series which are convenient in time. The same way is used to optimize the fuzzy model of time series in this study.

2. REVIEW OF FUZZY TIME SERIES STUDIES

Time series modeling using fuzzy inference maps every sample point in time series into fuzzy set and applies the “if-then” fuzzy rule. Wang (1997), Yu (2004) and Nurhayadi *et al*. (2012) have explained briefly that the formations of weighted fuzzy rules are as follows:

Step 1. Defining fuzzy set containing input and output spaces.

Given a pair of input-output (y_{t-p}, y_t), t = 1, 2, …., m where $(y_{t-p}, y_t) \in [\alpha,\beta]\times[\alpha,\beta] \subset R$, with y_{t-p} is the input and y_t is the output. In interval $[\alpha, \beta]$, defined a complete fuzzy sets A_k, k =1, 2, 3, … , q; where for every $y_{t-p} \in [\alpha,\beta]$ there exists A_k such that $\mu_{Ak}(y_{t-p}) \neq 0$ and $\mu_{Ak}(y_{t-p})$ is the membership degree of y_{t-p} in fuzzy set A_k.

Step 2. Generating the rule of every pair of input-output.

For every pair of input-output (y_{t-p}, y_t) if-then fuzzy rule is formed as follows:

$$if\ y_{t-p}\ is\ A^*)\ then\ \hat{y}_t = y_t$$

where, A^* is fuzzy set A_k which has the biggest $\mu_{Ak}(y_{t-p})$.

Step 3. Constructing of fuzzy rule base.

Group the rules generated in Step 2 based on the similar A^*. They are the rule’s components in part “if” and the consequence of $\bar{y}_k$ which is the mean of y_t is selected from part “then:

$$if\ (y_{t-p}\ is\ A_k)\ then\ \hat{y}_t = \bar{y}_t$$
$$k = 1, 2, \ldots, q$$

Basis of fuzzy rule has to satisfy three conditions below:

- In the rules generated in Step 2, there is no conflict between one rule and another, it means that the similar antecedents do not happen but the different consequences do
- The weights of basis elements are obtained from numbers of groups in Step 2
- Experiences of experts can be included into the basis of fuzzy rules

Step 4. Developing fuzzy system based on the basis of fuzzy rules

For example, obtain k basis of fuzzy rules, consider singleton fuzzifier, a multiplication in machine of fuzzy inference and defuzzifier of center mean, then use a formula below Equation 1:

$$\hat{y}_t = \frac{\sum_{k=1}^{q} w_k \bar{y}_k\, \mu_{A_k}(y_{t-p,})}{\sum_{k=1}^{q} w_k\, \mu_{A_k}(y_{t-p})} \quad (1)$$

Where:

$\hat{y}_t$ = Value estimation of y_t

w_k = Weight of fuzzy rule base

$\bar{y}_k$ = Part constant of part “then” and

$\mu_{Ak}(y_{t-p})$ = Membership degree of y_{t-p} in fuzzy set A_k

3. TRANSLATION OF FUZZY MODEL

There are many things which affect the accuracy of time series fuzzy model, among others are rounding numbers and choosing in consistent way. In order to increase the accuracy, the model needs to be given some advanced treatments. Nurhayadi *et al*. (2014) have done translation to minimize the Mean Absolute Error (MAE) The treatment discussed in this study is a model translation to reduce Mean Square Error (MSE). For example, let $\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_n$ be the values resulted in prediction using model (1) for set of time series values $y_1, y_2, \ldots y_n$ where Equation 2:

$$\hat{y}_1 = y_1 + e_1, \hat{y}_2 = y_2 + e_2, \ldots, \hat{y}_n = y_n + e_n \quad (2)$$

The modeling often yields more prediction points which are over than ones which are under estimate, or vice versa. Certainly, sliding the model slightly upward or downward can affect the error. By choosing movement that gives more error reduction, the MSE will be smaller. This sliding process is shown in **Fig. 1**.

Furthermore, if the translation is applied to the model in the extent of mean error directing to zero, i.e., $\hat{Y}^* = \hat{Y} - \bar{e}$, then MSE of predictors will reach the minimum value of translation.

Theorem 1

Let Y be the realization of time series, $\hat{Y}$ is prediction model, $\hat{Y}^* = \hat{Y} - \bar{e}$ where $\bar{e}$ is mean error and $e = \hat{Y} - Y$, then MSE($\hat{Y}^*$) $\leq$ MSE($\hat{Y}$).

Proof

If the model $\hat{Y}$ is slid in the extent of δ, i.e., $\hat{Y}^* = \hat{Y} + \delta$, then the function MSE($\hat{Y}^*$) is as follows:

$$MSE(\hat{Y}^*) \sum_{i=1}^{n} (e_i + \delta)^2$$

$$= \sum_{i=1}^{n} (e_i^2 + 2e_i\delta + \delta^2)$$

$$= \sum_{i=1}^{n} e_i^2 + 2\delta \sum_{i=1}^{n} e_i + n\delta^2$$

MSE($\hat{Y}^*$) is a quadratic function of δ. The value of this function will be minimum if the derivative to δ equals to zero, i.e:

$$\text{MSE}\frac{d\left(MSE(Y^*)\right)}{d\delta} = 0$$

$$2n\delta + 2\sum_{i=1}^{n} e_i = 0$$

$$\delta = -\frac{2\sum_{i=1}^{n} e_i}{2n} = -\frac{\sum_{i=1}^{n} e_i}{n} = -\bar{e}$$

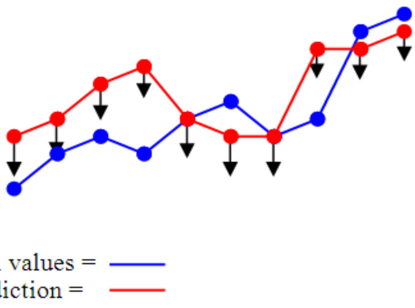

Fig. 1. Model sliding to reduce MSE

So that:

$$MSE(\hat{Y}) \sum_{i=1}^{n} e_i^2$$

$$= \sum_{i=1}^{n} (e_i + 0)^2 \geq \sum_{i=1}^{n} (e_i - \bar{e})^2 = MSE(\hat{Y}^*)$$

Based on Theorem 1, if the fuzzy model of time series data (1) is translated in the extent of $-\bar{e}$ into Equation 3:

$$\hat{y}_t = \frac{\sum_{k=1}^{q} w_k \bar{y}_k \mu_{A_k}(y_{t-p})}{\sum_{k=1}^{q} w_k \mu_{A_k}(y_{t-p})} - \bar{e} \quad (3)$$

Then it is expected to be able to reduce MSE.

4. VERIFICATIONS AND COMPARISONS

Ion the method discussed in this study will be compared to other methods. The data used is the enrollment data of Alabama University because this data is widely used by many researchers in time series forecasting.

The first differences of the of enrollment data of Alabama University is normalized using the formula below Equation 4:

$$z_t = \frac{y'_t - \bar{y}}{s} \quad (4)$$

y'_t = First differences of the data
$\bar{y}$ = Mean
s = Standard deviation

The results of normalization in column 4 of **Table 1** are applied in the rules at section 2, using $p = 1$ and $q = 9$.

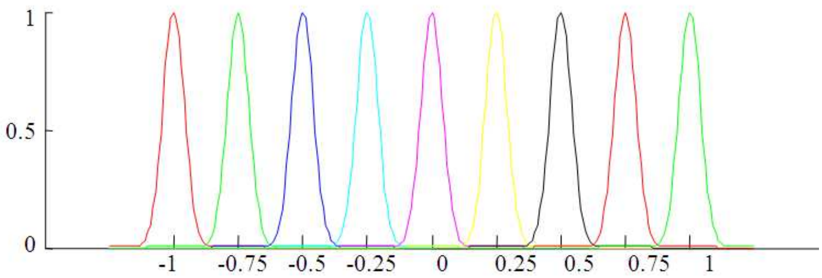

Fig. 2. The fuzzy sets with gaussian membership functions

Step 1. Create 9 fuzzy sets as **Fig. 2**, with Gaussian membership functions Equation 5:

$$\mu_{A_k}(z_{t-p}) = \exp\left(-\frac{1}{2}\left(\frac{z_{t-p}-c_k}{s}\right)^2\right) \tag{5}$$

k = 1, 2, … , q
c_k = -1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 1
s = (q-1)/3

Step 2. Make the fuzzy rules based on the Fuzzy sets in Step-1 and the data in column 4 of **Table 1.**

The third column in **Table 2** contains the antecedents, they are the part of IF rule. While in the fourth column, there are also consequents which are part of THEN rule.

Step 3. Make the basis of fuzzy rules by grouping rules based on the similar fuzzy sets and consequently the mean values are taken for every group. The weight w_j is given based on the numbers of rules which arrange it:

Rule 1: If z_{t-1} is A_1 then $\bar{z}_t$ = -0.9943 weight = 3
Rule 2: If z_{t-1} is A_2 then $\bar{z}_t$ = 0.0259 weight = 1
Rule 3: If z_{t-1} is A_3 then $\bar{z}_t$ = -0.1701 weight = 2
Rule 4: If z_{t-1} is A_4 then $\bar{z}_t$ = -1.2577 weight = 2
Rule 5: If z_{t-1} is A_5 then $\bar{z}_t$ = 0.7008 weight = 3
Rule 6: If z_{t-1} is A_6 then $\bar{z}_t$ = -0.4693 weight = 1
Rule 7: If z_{t-1} is A_7 then $\bar{z}_t$ = 0.0469 weight = 1
Rule 8: If z_{t-1} is A_8 then $\bar{z}_t$ = -0.7458 weight = 1
Rule 9: If z_{t-1} is A_9 then $\bar{z}_t$ = 0.7457 weight = 6

The prediction is done by substituting z_{t-1} and $\bar{z}_t$ in Rule 1 until Rule 9 to the formula (1). The result $\hat{z}_t$ is shown in Column 3 of **Table 3**.

Because the input of the process uses the normalized first differences of the data, then the results of the prediction must be converted using the invers of formula (4) and the invers of the first differentiation. These results are shown in column 4 of **Table 4**. In the same time, the translation using formula (3) results in prediction values shown in column 5.

From the comparison of the prediction results in column 4 and 5 with the actual values in column 6, we can see that the translation in the extent of mean error can decrease the MSE.

Table 1. The actual enrollment data, difference-1 and normalization

Year t	Actual y_t	1st difference y'_t	Norm z_t
1971	13,055	508	0.4039
1972	13,563	304	0.0469
1973	13,867	829	0.9656
1974	14,696	764	0.8519
1975	15,460	-149	-0.7458
1976	15,311	292	0.0259
1977	15,603	258	-0.0336
1978	15,861	946	1.1704
1979	16,807	112	-0.2891
1980	16,919	-531	-1.4143
1981	16,388	-955	-2.1562
1982	15,433	64	-0.3731
1983	15,497	-352	-1.1010
1984	15,145	18	-0.4536
1985	15,163	821	0.9516
1986	15,984	875	1.0461
1987	16,859	1291	1.7741
1988	18,150	820	0.9499
1989	18,970	358	0.1414
1990	19,328	9	-0.4693
1991	19,337	-461	-1.2918
1992	18,876		

The results of the forecast based on the proposed models, compared with the results Song and Chissom (1993), Sullivan and Woodall (1994), Chen (1996) and Cheng *et al*. (2008) are shown in **Table 5.**

Table 2. Fuzzy rule

t-1	Z_{t-1}	Fuzzy set	Z_t
1971	0.4039	A7	0.0469
1972	0.0469	A5	0.9656
1973	0.9656	A9	0.8519
1974	0.8519	A8	-0.7458
1975	-0.7458	A2	0.0259
1976	0.0259	A5	-0.0336
1977	-0.0336	A5	1.1704
1978	1.1704	A9	-0.2891
1979	-0.2891	A4	-1.4143
1980	-1.4143	A1	-2.1562
1981	-2.1562	A1	-0.3731
1982	-0.3731	A4	-1.1010
1983	-1.1010	A1	-0.4536
1984	-0.4536	A3	0.9516
1985	0.9516	A9	1.0461
1986	1.0461	A9	1.7741
1987	1.7741	A9	0.9499
1988	0.9499	A9	0.1414
1989	0.1414	A6	-0.4693
1990	-0.4693	A3	-1.2918

In **Fig. 3**, the graph of actual values is indicated by the blue plot and the graph of predictions is indicated by the red one.

Based on the results from **Table 5**, the proposed model has a smaller Root Mean Square Error (RMSE) and less average error than the other models.

Table 3. The predictions using weighted fuzzy model

t-1	Z_{t-t}	$\hat{z}_t$
1971	0.4039	0.0390
1972	0.0469	0.7008
1973	0.9656	0.7457
1974	0.8519	-0.4820
1975	-0.7458	0.0259
1976	0.0259	0.7008
1977	-0.0336	0.7008
1978	1.1704	0.7457
1979	-0.2891	-1.2576
1980	-1.4143	-0.9943
1981	-2.1562	-0.9943
1982	-0.3731	-0.7891
1983	-1.1010	-0.9943
1984	-0.4536	-0.1701
1985	0.9516	0.7457
1986	1.0461	0.7457
1987	1.7741	0.7457
1988	0.9499	0.7457
1989	0.1414	-0.2116
1990	-0.4693	-0.1701

Table 4. The results of enrollment prediction

t	$\hat{z}_t$	$\hat{y}'_t$	$\hat{y}_t$	$\hat{y}^*_t$	y_t
1972	0.039	299.4726			
1973	0.7008	677.6638	13,862	13,834	13,867
1974	0.7457	703.333	14,545	14,517	14,696
1975	-0.482	1.7408	15,399	15,371	15,460
1976	0.0259	292	15,462	15,434	15,311
1977	0.7008	677.6665	15,603	15,575	15,603
1978	0.7008	677.6652	16,281	16,253	15,861
1979	0.7457	703.3333	16,539	16,511	16,807
1980	-1.2576	-441.4974	17,510	17,482	16,919
1981	-0.9943	-291	16,478	16,450	16,388
1982	-0.9943	-291	16,097	16,069	15,433
1983	-0.7891	-173.7338	15,142	15,114	15,497
1984	-0.9943	-291	15,323	15,295	15,145
1985	-0.1701	179.9924	14,854	14,826	15,163
1986	0.7457	703.331	15,343	15,315	15,984
1987	0.7457	703.3333	16,687	16,659	16,859
1988	0.7457	703.3333	17,562	17,534	18,150
1989	0.7457	703.3304	18,853	18,825	18,970
1990	-0.2116	156.2444	19,673	19,645	19,328
1991	-0.1701	179.9992	19,484	19,456	19,337
1992			19,517	19,489	18,876
Mean			28	0	
MSE			135023	134238	
RMSE			367	366	

Table 5. Comparisons of the prediction results for enrollment with different models

Year	Actual	Song (1993a; 1993b)	Sullivan and Woodall (1994)	Chen (1996)	Cheng *et al*. (2008)	Proposed method
1971	13,055					
1972	13,563	14,000	13,500	14,000	13,680.5	
1973	13,867	14,000	14,500	14,000	13,731.3	13,834
1974	14,696	14,000	14,500	14,000	13,761.2	14,517
1975	15,460	15,500	15,231	15,500	15,194.6	15,371
1976	15,311	16,000	15,563	16,000	15,374.8	15,434
1977	15,603	16,000	15,500	16,000	15,359.9	15,575
1978	15,861	16,000	15,500	16,000	16,410.3	16,253
1979	16,807	16,000	15,500	16,000	16,436.1	16,511
1980	16,919	16,813	16,684	16,833	17,130.7	17,482
1981	16,388	16,813	16,684	16,833	17,141.9	16,450
1982	15,433	16,789	15,500	16,833	15,363.8	16,069
1983	15,497	16,000	15,563	16,000	15,372.1	15,114
1984	15,145	16,000	15,563	16,000	15,378.5	15,295
1985	15,163	16,000	15,563	16,000	15,343.3	14,826
1986	15,984	16,000	15,563	16,000	15,345.1	15,315
1987	16,859	16,000	15,500	16,000	16,448.4	16,659
1988	18,150	16,813	16,577	16,833	17,135.9	17,534
1989	18,970	19,000	19,500	19,000	18,915.0	18,825
1990	19,328	19,000	19,500	19,000	18,997.0	19,645
1991	19,337	19,000	19,500	19,000	19,032.8	19,456
1992	18,876		19,000	19,000	19,033.7	19,489
RMSE		650	621	638	438	366

Table 6. Actual and forecasting with translation results for the enrollment

Year	Actual	Song- $\bar{e}$	Sullivan- $\bar{e}$	Chen- $\bar{e}$	Cheng- $\bar{e}$
1971	13,055				
1972	13,563	13,930.7	13,642.5	13,923.1	13,805.7
1973	13,867	13,930.7	14,642.5	13,923.1	13,856.5
1974	14,696	13,930.7	14,642.5	13,923.1	13,886.9
1975	15,460	15,430.7	15,373.5	15,423.1	15,319.8
1976	15,311	15,930.7	15,705.5	15,923.1	15,500.0
1977	15,603	15,930.7	15,642.5	15,923.1	15,485.1
1978	15,861	15,930.7	15,642.5	15,923.1	16,535.5
1979	16,807	15,930.7	15,642.5	15,923.1	16,561.3
1980	16,919	16,743.7	16,826.5	16,756.1	17,255.9
1981	16,388	16,743.7	16,826.5	16,756.1	17,267.1
1982	15,433	16,719.7	15,642.5	16,756.1	15,489.0
1983	15,497	15,930.7	15,705.5	15,923.1	15,497.3
1984	15,145	15,930.7	15,705.5	15,923.1	15,503.7
1985	15,163	15,930.7	15,705.5	15,923.1	15,468.5
1986	15,984	15,930.7	15,705.5	15,923.1	15,470.3
1987	16,859	15,930.7	15,642.5	15,923.1	16,573.6
1988	18,150	16,743.7	16,719.5	16,756.1	17,261.1
1989	18,970	18,930.7	19,642.5	18,923.1	19,040.2
1990	19,328	18,930.7	19,642.5	18,923.1	19,122.2
1991	19,337	18,930.7	19,642.5	18,923.1	19,158.0
1992	18,876			18,923.1	19,158.9
RMSE		646.7	605.1	633.7	420

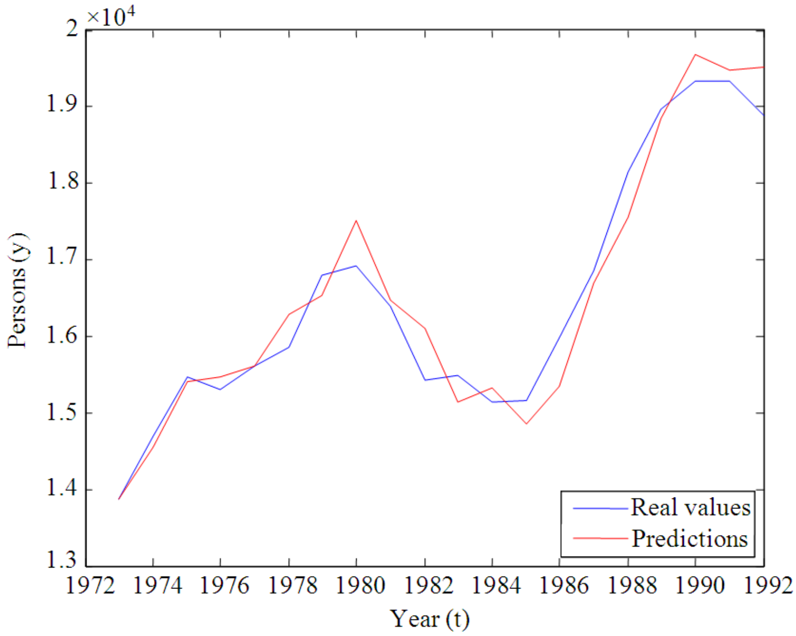

Fig. 3. The actual value of the enrollment and prediction results

If the translation is applied to models generated by other researchers, then the RMSE in each model will also decline and it means that the MSE declines too. This result is shown in **Table 6**.

Based on the results from **Table 6**, the translation of model can reduce the forecasting done by other researchers.

5. CONCLUSION

Theoretically, a translation in the extent of mean error can reduce MSE. The translation in the extent of mean error applied to model (1) that uses Gaussian membership function gives a better result than the other models and it is shown in **Table 5**. If this translation is applied to each of other models, then the RMSE of each model will also decline.

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