Review

Rental Estimation of Reverse Mortgages (RM): The Case of Home as Rental in Retirement

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Abstract: The aim of this paper was to compare the rental market conditions in retirement with an RM contract that involves a retiree's home and its valuation. It is noted that the current rental market conditions are to be examined thoroughly before any engagement in RM is to be instituted. The rental value per week for 1, 2-Bedroom Units and 3 or more Bedroom homes are to study in some depth. The availability of rental housing should also be studied to determine whether it is indeed possible to obtain secure longer term rental housing for the retired persons. In this study only a certain group has been investigated and further work is necessary if RM is to be a major contract that retired persons may engage in thus achieving a better quality of life after retirement given the many years of service couples have given to their country. The lack of engagement in Australia is a two way story in that the less than usual interest of retiree's on RM is met by the banks very low level of advertising of the same. The lack of information has created a lack of trust in RM; the thought of letting someone else own apart of one's home towards the later part of one's life in some manner is not well accepted or understood given the retiree's home is mostly a mortgage free asset. The modelling and analyses conducted in this study show that the return in terms of rental is well in competition with the existing rental market in Australia and in particular when using the largest city data from Sydney and other rental markets.

Keywords: Rental Housing Market, Reverse Mortgages, Retired Persons, Comparative Analysis

Introduction

The financial instrument which provides a vehicle of conversion of home equity into cash is mostly referred to as a Reverse mortgage (Edward *et al*., 2007; Nance, 2016). The process allows mostly the elderly to use the equity in their home to borrow funds when needed (Bridge *et al*., 2010). So the homeowner converts their equity into cash to remain or stay at the property as long as they please after a certain age say 62 for example. The house is in essence enabling the lender to rent back from the borrower. Essentially, then a reverse mortgage is to borrow against the value of the owner's property (Nance, 2016; Warshawsky, 2017; Warshawsky and Zohrabyan, 2016). One of the possible borrowings (as described in the next section) is that the homeowner receives a monthly amount over the time of the so called loan in reverse; essentially then this is an annuity, based on the

value of his property (Bridge *et al*., 2010; Nance, 2016). The reverse mortgaged amount in then terminate with accumulated interest until the due date or the death of the owner, at which time the lender may sell the property to obtain the smaller of the debt incurred or the net value of the property. The remaining value of the property is passed onto the deceased's estate (Liang *et al*., 2008).

There two main issues that concern the borrower and the lender in this reverse mortgage game plan. (i) a growing economy grows leads to higher property values over the years thus the homeowner/borrower will have a more valuable asset against which he/she can continue to utilize the excess home equity; (ii) there could also be a benefit to the lender in that with appreciating property values over time would reduce the chance of the total debt exceeding the property valuation at the conclusion or termination time (Firman, 2015).

There are many countries now who are particularly investigating the concept of reverse mortgage such as Singapore (Cho *et al*., 2013). The situation is similar in the US and UK where the concept of reverse mortgages is gaining acceptance (Chen *et al*., 2010). The Singapore government is concerned with their ageing population as the percentage of the elderly population grows in numbers. Just as in the US and UK, there is now a concern regarding the elderly's comfortability of living given that they generally have much equity tied up in their home. As such the concept of reverse mortgages among countries that continue to have a growing number in the group of elderly population is gaining acceptance and growing; such as Australia (Reed, 2009).

Background to Reverse Mortgage (RM)

In developed countries growing proportion of elderly in the communities are finding that the regular income from savings and/or pensions are usually insufficient for a comfortable level of living life, yet most elderly live in houses which are almost free of mortgages (Firman, 2015). The most important aspect is that most of the elderly who are in such a situation would not prefer to sell their houses and then move to another environment either downsizing or into care communities for example, until they are unable to totally take care of themselves (Thomas, 2014a; 2014b).

In response of this aspect of financial demand, the financial agencies and markets in many counties have developed financial products to provide benefits needed in retirement but also to help to manage the associated risks involved (Maestas and Zissimopoulos, 2010). There are many financial products but there are two main kinds of products now available to support the elderly and these are called Reverse Mortgages (RM) and Home Reversions (HR) (Alai *et al*., 2014). By far the most common is the Reverse Mortgages (RM). The major attraction of RM is that the borrower makes no payments until the RM is terminated; either by death of the borrower or the borrower moving out of home or selling the home. The ownership is retained by the homeowner throughout therefore the owner may choose to sell the house at a time but then the seller/s must pay the total debts incurred in RM to that time. Reverse mortgages can also be issued couples and such cases the lender uses the age of the younger of the couple for mortgage consideration (Nakajima, 2014a; 2014b).

As the borrowing in Reverse Mortgage is based on the equity of the borrower's house, there is the risk that the value of the house at termination is insufficient to cover the liability and when this occurs lender may not have recourse to other avenues to extract funds. The RM with such non-recourse guarantee for the shortfall are said to have No Negative Equity Guarantee (NNEG). In particular US RM's are administered by the Housing and Urban Development (HUD) and the NNEG and any shortfall of funds at the termination are covered by the Federal Government. Some qualitative and quantitate description about in relation to Reverse Mortgages in US are available in Nakajima (2012). In other countries, the lender may likely accommodate the value or risk related to NNEG within the RM design.

The main aim of this paper is to examine the concept of free rental in Reverse Mortgages and how this is incorporated into the NNEG component. Essentially, the goal here is to estimate the free rental value of the program given the known parameters in RM.

Generally, those who undertake Reverse Mortgage are usually over 60 (62 in US); and the borrowing is generally limited to a fraction of the appraised value of the house at the time of borrowing. In many cases, the owners lack the mathematics of the financial calculations involved (Hopf and Tularam, 2014; Tularam, 2013). However, the borrower can choose one of the six possible alternatives within the program:

1. A lump-sum: Borrower receives a single lump-sum payment
2. Tenure: borrower receives a fixed periodic payment as long as at least one of the borrowers continue to live in the same house
3. Term: Borrower receives a fixed payment for a fixed length of time
4. Line of Credit (LOC): Borrower can flexibly draw cash, up to a limit, during a predetermined drawing period
5. Modified tenure: Combination of tenure and LOC
6. Modified term: Combination of term and LOC

Whatever the type of borrowing, the lender will charge an initial and regular premiums to mitigate the risks of NNEG and/or some failure of the program but the borrower stays in the home free of any rental until the termination due to death or moving out of the house. The borrower has to incur all maintenance expenses for the property until the policy termination

Modelling RM using Lump Sum Model Using Continual Time Case

Consider a retired individual who takes out a reverse mortgage loan amount of A_0 dollars, against his house currently valued at H_0 dollars. If at time t the loan amount is A_t, the house price is H_t and the cost of the capital is C_t, then the value of the reverse mortgage loan is labelled V_t, which is equal to min (A_t, H_t). In this manner, the loss to the lender given as L_t is: $L_t = C_t - V_t = C_t - $ min (A_t, H_t).

If the loan amount A_t accumulates at a risk free interest rate of r_t plus a risk premium rate of λ, while house price H_t appreciates at a rate of δ_t; and the cost of the capital C_t accumulates at an interest rate of γ, then the loan value process can be given as:

$$A_t = A_0 \exp(\int_0^t (r_s + \lambda)\, ds),$$

and the house price process is:

$$H_t = H_0 \exp\left(\int_0^t (\delta_s)\, ds \right),$$

and the cost of capital process is given by:

$$C_t = A_0 \exp\left(\int_0^t (\gamma_s)\, ds \right).$$

Since the value of the loan repayment V_t is the smaller of the house price and the accumulated loan amount, we have:

$$V_t = \min\left(A_0 \exp\left(\int_0^t (r_s + \lambda)\, ds \right), H_0 \exp\left(\int_0^t (\delta_s)\, ds \right) \right).$$

The loss to the lender L_t at time t can be given as:

$$L_t = A_0 \exp\left(\int_0^t (\gamma_s)\, ds \right) - \min\left[\begin{array}{c} A_0 \exp\left(\int_0^t (r_s + \lambda)\, ds \right), \\ H_0 \exp\left(\int_0^t (\delta_s)\, ds \right) \end{array} \right].$$

The time of the reverse mortgage loan is a random variable T; which is the future remaining lifetime of the borrower. At settlement the loss L_T to the lender can be expressed as the random variable:

$$L_T = A_0 \exp\left(\int_0^T (\gamma_s)\, ds \right) - \min\left[\begin{array}{c} A_0 \exp\left(\int_0^T (r_s + \lambda)\, ds \right), \\ H_0 \exp\left(\int_0^T (\delta_s)\, ds \right) \end{array} \right]. \quad (1)$$

Using the actuarial equivalence principle that states that the present value of total expected gain ought to be the same value as the present value of total expected loss; therefore we can state: $E[e^{-rT} L_T] = 0$.

Using this fact, the pricing equation for the reverse mortgage can be written as follows:

If $E[e^{-rT} L_T] = 0$ then:

$$E\left[A_0 \exp\left(\int_0^T (\gamma_s)\, ds \right) \right] = E\left[\min\left[\begin{array}{c} A_0 \exp\left(\int_0^T (r_s + \lambda)\, ds \right), \\ H_0 \exp\left(\int_0^T (\delta_s)\, ds \right) \end{array} \right] \right] \quad (2)$$

Using the previous equation if a certain level of risk premium λ is used then solving the above can provide the maximum loan amount A_0 that can be given; while in reverse if we are given an initial loan amount A_0, then a fair risk premium λ that the lender should use can be determined.

Given the above background to modelling RM, we can return to the main goal of this paper that is to estimate the free rental value of the program using the known parameters. In this study, we use the termination distribution estimated from US data on Reverse Mortgages (Dayananda *et al.*, 2015), which shown to follow a Weibull distribution. The following section develops the models to estimate the free rental for the borrower under the reverse mortgage of the form: (i) lump-sum RM and (ii) tenure RM. The models are then utilized to estimate the free rental values for given plausible parameters used in the industry.

Estimation the Free Rentals - Lump-Sum RM

We define the following:

$H(t)$ = The value of the house price process at time t, so $H(0)$ is the initial value

L_t or L_0 = The loan value process at time t and the loan value at t = 0 respectively

k_1 = The premium rate on the loan balance charged after the initial premium k_0

$H(0)$ = The initial premium cost of the reverse mortgage

r = he risk-free rate assumed constant

r_1 = The interest rate charged by the lender

T = The termination time of the reverse mortgage policy

$H(T)$ = The Termination value of the house

$U(t)$ = The payoff at time t:

$U(T)$ = The payoff at the termination time T

λ = The fraction of the house value that is not be included in the RM; and

$1-\lambda$ = The fraction of house price value considered for reverse mortgage

Since only a fraction of the house value is involved in the RM, the borrower's payoff from RM at time T may be written as: $U(T) = ((1-\lambda)H(T) - L_0 \exp\{(r_1 + k_1)T\})_+$, where, $U_+ = \text{Max}(U, 0)$. The first term on the right is the final value of the house ($H(T)$) while the second term on the right is total debts including the lender's cost of capital and risk insurance rate. The $U(T)$ is only a positive function [(Max $(U, 0)$]; because if the loan increases beyond the $H(T)$ value the liability is accepted by the lender (Not the borrower so this is the lender's risk of loss at Termination).

There are number of costs involved in RM such as insurance cost including other costs to the borrower that can now be incorporated into RM as the Total Costs at time T:

$$Total\ Costs\ = k_0 H(0)\exp(rT)$$
$$+k_1 \exp(rT)\{\exp(r_1 - r + k_1)T - 1\} / (r_1 - r + k_1). \quad (3)$$

The first term on the right is the total cost the borrower that is a fraction (k_0) of $H(0)$; that is paid upfront as a premium for costs of setting up an RM; the second on the right is the future value of (T-1) payments of the lender's rate of capital (r_1) plus insurance or risk premium of RM from time $t = 2$; k_1 is the rate that accounts for the risk insurance costs of RM over the time of the loan.

The final gain or loss in gross terms to the borrower at time T can be written as:

$$Loss\ or\ Gani = H(T) - L_0 \exp(rT)$$

where, L_0 is the initial lump sum loan amount from the lender as it grows over time to T; allowing for the costs, the net gain/loss to the borrower at termination time T (G_1) may be written as:

$$G_1 = ((1-\lambda)H(T) - L_0 \exp\{(r_1 + k_1)T\})_+$$
$$+\lambda H(T) - k_0 H(0)\exp(rT)$$
$$-k_1 \exp(rT)\{\exp((r_1 - r + k_1)T) - 1\} \quad (4)$$
$$/(r_1 - r + k_1) - (H(T) - L_0 \exp(rT)).$$

Thus the expected risk-neutral present value of the net gain/loss to borrowers:

$$E_Q\{G_1 e^{-rT}\} = E_Q[e^{-rT}((1-\lambda)H(T)$$
$$-L_0 \exp((r_1 + k_1)T))_+] + \lambda H(0)$$
$$-k_0 H(0) - k_1\{\exp((r_1 - r + k_1)T) - 1\} / (r_1 - r + k_1) \quad (5)$$
$$-H(0) + L_0.$$

We assume that the house price process $H(t)$ has a geometric Brownian motion with mean and variance μ and σ^2 respectively. Then using Black and Scholes (1973), the first component of (5) reduces to:

$$E_Q[((1-\lambda)H(T) - L_0 \exp\{(r_1 + k_1)T\})_+$$
$$= (1-\lambda)\{H(0)\Phi(d_1) - \exp(-rT)K_1(T)\Phi(d_2)\} \quad (6)$$

with Φ as the cumulative distribution function for the standard normal distribution:

$$d_1 = \{\ln(H(0) / K_1(T)) + (r + 0.5\sigma^2)T\} / \sigma\sqrt{T},$$
$$K_1(T) = L_0 \exp((r_1 + k_1)T) / (1-\lambda)$$
$$and\ d_2 = d_1 - \sigma\sqrt{T}.$$

Thus we can express the relation in (5) for given T as:

$$E\{G_1 e^{-rT}T\}$$
$$E\{Ge + \lambda H(0) - k_0 H(0)$$
$$-k_1\{\exp((r_1 - r + k_1)T) - 1\} / (r_1 - r + k_1) \quad (7)$$
$$-H(0) + L_0$$

The expression in (5) provides the borrower, provided it is again, the expected present value of the rental free residence. Let R_1 = Rental rate to the borrower corresponding to the reverse mortgage policy. Then the present value of the rental for the period T:

$$R_1 \overline{a}_{\overline{T}|} = R\ (1 - e^{-r_2 T}) / r_2 \quad (8)$$

where, r_2 = Interest rate applicable to rental policies in the period (0, T).

Hence:

$$E_Q\{Ge^{-rT}\} = E\{R(1 - e^{-r_2 T}) / r_2\}. \quad (9)$$

The relation in (8) establishes a mechanism to evaluate the free rental rate associated with the reverse mortgage policy. It is given by:

$$R_1 = \frac{r_2 E\{Ge^{-rT}\}}{E\{1 - e^{-r_2 T}\}}. \quad (10)$$

The distribution for the termination time T was found to be Weibull with density $f(t)$, Dayananda *et al.* (2016), unpublished and is given by:

$$f(t) = \alpha t^{\alpha-1} \exp\{-(t / \beta)^\alpha\} / \beta^\alpha. \quad (11)$$

Thus the expectations needed in (10) reduce to:

$$E\{G_1 e^{-rT}\} = \int_0^{105-x} G_1 e^{-rt} f_1(t)\, dt,$$
$$and \quad (12)$$
$$E\{1 - e^{-r_2 T}\} = \int_0^{105-x} (1 - e^{-r_2 t}) f_1(t)\, dt$$

where, $f_1(t) = f(t) / \int_0^{105-x} f(t)\, dt$ and x is the age of the reverse mortgage borrower and 105 is the extreme age in life.

Case 2 Tenure: Regular Payment to the Reverse Mortgage Owner

Let Y be the continuous payment to the borrower whose age is x under tenure policy:

$$Y a_{\overline{105-x}|} = (1-\lambda)H(0) \tag{13}$$

assuming that the reverse mortgage is based on total house equity value.

Thus at termination time T, the value of the payments is accumulated to:

$$Y s_{\overline{T}|} \text{ at rate } r_1 + k_1. \tag{14}$$

Now the components for the computation of the net value to the borrower at termination time T are:

(1) Payoff and the residual home value $((1-\lambda) H(T) - Y s_{\overline{T}|})_+ + \lambda H(T)$,

(2) Insurance cost:

$$k_0 H(0) + k_1 \frac{Y e^{(r_1+k_1)T}}{(r_1+k_1)} \left[\begin{Bmatrix} \frac{1}{r} + \frac{e^{-(r+k_1)T}}{(r-r+k_1)} \end{Bmatrix} \\ - \begin{Bmatrix} \frac{1}{r} + \frac{1}{(r_1-r+k_1)} \end{Bmatrix} e^{-rT} \end{Bmatrix} \right], \quad (15)$$

(3) Termination proceeds $H(T) + Y a_{\overline{T}|} e^{rT}$.

Therefore the total net benefit to the borrower at termination time T is:

$$G_2 = ((1-\lambda)H(T) - Y s_{\overline{T}|})_+ - (1-\lambda)H(T) - k_0 H(0)e^{rT} - $$
$$k_1 \frac{Y e^{(r+k_1)T}}{(r_1+k_1)} [\{\frac{1}{r} + \frac{e^{-(r_1+k_1)T}}{(r-r+k)}\} - \{\frac{1}{r} + \frac{1}{(r-r+k_1)}\}e^{rT}] \tag{16}$$

Thus the present value of the net borrower benefit is:

$$E_Q(G_2 e^{-rT}) = E_Q[e^{-rT}(((1-\lambda)$$
$$H(T) - Y s_{\overline{T}|})_+) - (1-\lambda)H(0) - k_0 H(0) - \tag{17}$$
$$k_1 \frac{Y e^{(r+k_1-r)T}}{(r_1+k_1)}[\{\frac{1}{r} + \frac{e^{-(r_1+k_1)T}}{(r-r+k)}\} - \{\frac{1}{r} + \frac{1}{(r-r+k_1)}\}e^{rT}]$$

Now following the same argument as in (9), the continuous free rental provided by the reverse mortgage is:

$$R = \frac{r_2 E[G_2 e^{-rT}]}{E[(1-e^{-r_2 T})]}. \tag{18}$$

Discussion of Numerical Results

The annual rental calculations provided in the figures and tables in this section are based on the formulas presented in the previous section. The integrations were performed numerically and the results are presented in the Tables 1-4. The annual rental charge allowed using the formula of various set of couples was investigated for three different levels of risk (σ); namely, 0.1, 0.2 and 0.3. The differing initial house valuations were also taken into account in that the real estate concerning average to upper middle level of dwellings were considered in the modelling; namely, 200K, 300K, 400K, 500K and 600K. Such values are line with those in the house valuation literature for the most Australian cities (see: https://www.ahuri.edu.au/).

A number of constants were used in the modelling such as Dayananda (2015) values of k_0, k_1, r, r_1 and r_2. The following constants were used to determine the parallel annual rental charges: namely:

$$k_0 = 0.02, \ k_1 = 0.005, \ r = 0.02, r_1 = 0.035, r_2 = 0.02$$

Similar tables can be developed to each age group for each risk value of sigma. Tables 1-3 above show various scenarios/cases of aged couples retiring but choosing to stay and live in their homes (with homes values from the range of 200K to 600K) as if they were in fact renting it from the bank; that is, the given the couples chose to take a reverse mortgage condition with institutions such as a bank that offers the same. The initial housing values being in line with the reverse mortgage values at the initial point; clearly, these are assessed by valuations of houses conducted by those institutions/companies that sell mortgage products such as the banks. Table 4 summarizes other tables of age couple but using a middling risk factor of 0.2 for sigma.

The Fig. 1 shows various groups plotted with Series 1 referring to the 65, 65 couple to Series 4: 75, 75 couple respectively. The first four columns of Table 3 are plotted and compared marginally in Fig 3. Series 1 is drawn for the couple aged 65, 65. The yearly rental ranges from 6313 to 18939 depending on the valuation of their house; varying from 200K to 600K respectively. This group would on the whole involve a somewhat higher risk to the bankers than others with the possibility of being healthier and living longer, thus renting for longer period so to say. We note a restriction in amount that is available for them column wise in Table 3 when compared to other columns. The Series 2 refers to the 65, 75 aged couple; while Series 3 refers to 65, 85 group; and finally the Series 4 is drawn for the 75, 85 group. A similar pattern is noted in each condition.

Interestingly, the most risky for the banks will be the higher aged couples but the amount of time to end point of their lives may be much less. So the total risk of paying out large amounts is less; and also therefore the rental amount yearly calculated will tend to be more than earlier year couples accordingly. The virtual rental yearly amounts is higher for the given house values for the age 75, 85 and 85, 85 couples when compared to others; others of lower age groups even when we compare the higher values housing (H(0) in the first column of Table 3); when allowing for the house valuations and examining marginally again, we see across rows, the values are increasing Fig. 3 and Table 3.

Figures 1, 2 and 3, the Y axis is the rental calculated in dollars yearly using our modelling procedure and it is noted that all are rising for all groups as ages of couples increase across rows of Table 3 (Fig. 1-3). The amounts are also increasing column wise since the bank housing valuation increases with higher real market values of homes (Fig. 1 and 2). In the end this may suggest that it is not appropriate to undertake a RM contract at higher age groups, for one can rent at lower levels by obtaining a rental condition in the market place for a similar house. However this analysis is presented and discussed later when the housing and rental market conditions are further analysed.

Table 1: Annual rental charge ($) for various initial house values - couples (Ages pair 65, 65)

H(0)	σ = 0.1	σ = 0.2	σ = 0.3
200000	6318	6313	6106
300000	9477	9470	9159
400000	12636	12626	12212
500000	15796	15783	15265
600000	18955	18939	18313

Table 2: Annual rental charge ($) for various initial house values - couples (Ages pair 65, 75)

H(0)	σ = 0.1	σ = 0.2	σ = 0.3
200000	6784	6780	6607
300000	10176	10170	9101
400000	13568	13561	13204
500000	16960	16951	16508
600000	20353	20341	19822

Table 3: Annual rental charge ($) for various initial house values - couples (Ages pair 65, 85)

H(0)	σ = 0.1	σ = 0.2	σ = 0.3
200000	7179	7175	7009
300000	10769	10763	10513
400000	14358	14351	14018
500000	17948	17938	17522
600000	21538	21526	21027

Table 4: Annual Rental charge ($) across couples of different age groups when σ = 0.2

H(0)	Aged couple					
	65,65	65,75	65,85	75,85	75,75	85,85
200000	6313	6780	7175	8057	7379	9397
300000	9470	10170	10763	12075	11069	14089
400000	12626	13561	14361	16100	14758	18785
500000	15783	16957	17938	20125	18448	23481
600000	18939	20341	21526	24150	22138	28177

Table 4: Annual Rental ($) with increasing ages of couples and increases in housing valuations ($)

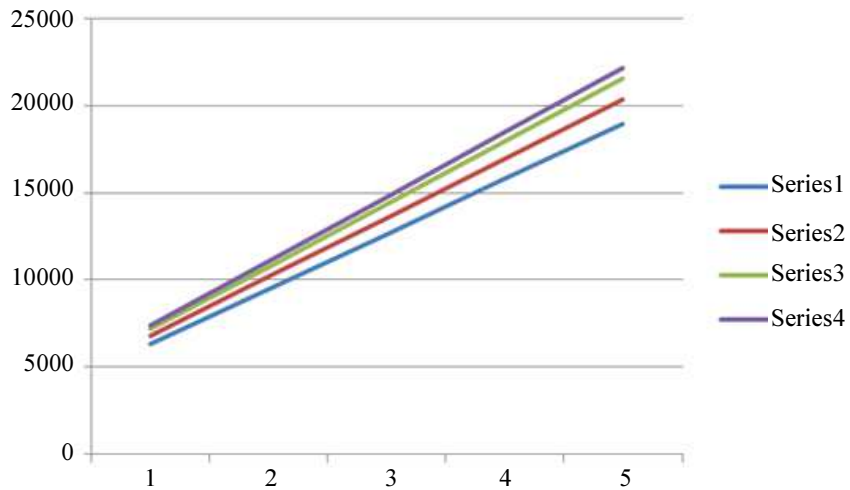| H(0)/Age	65, 65	65, 75	65, 85	75, 75	75, 85	85, 85
200000	6313	6780	7175	7379	8057	9397
300000	9470	10170	10763	11069	12075	14089
400000	12626	13561	14361	14758	16100	18785
500000	15783	16957	17938	18448	20125	23481
600000	18939	20341	21526	22138	24150	28177

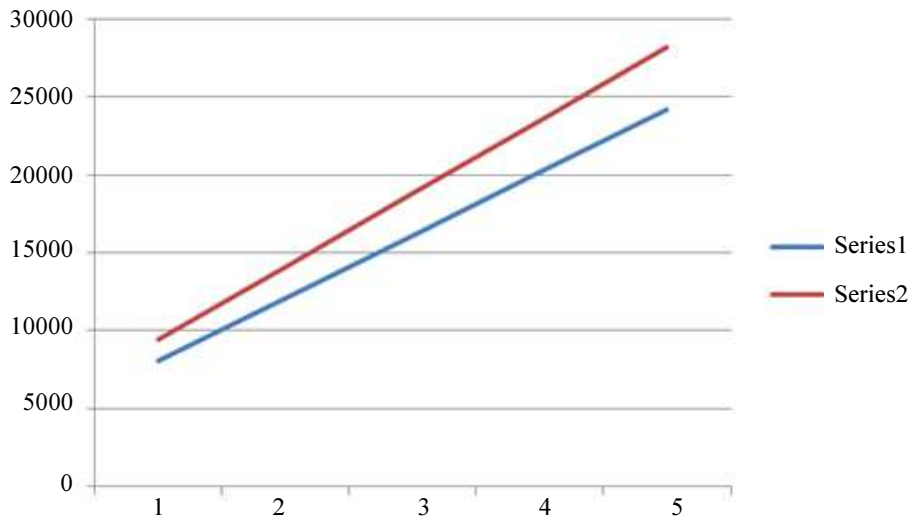**Fig. 1:** Plots of columns of Table 3

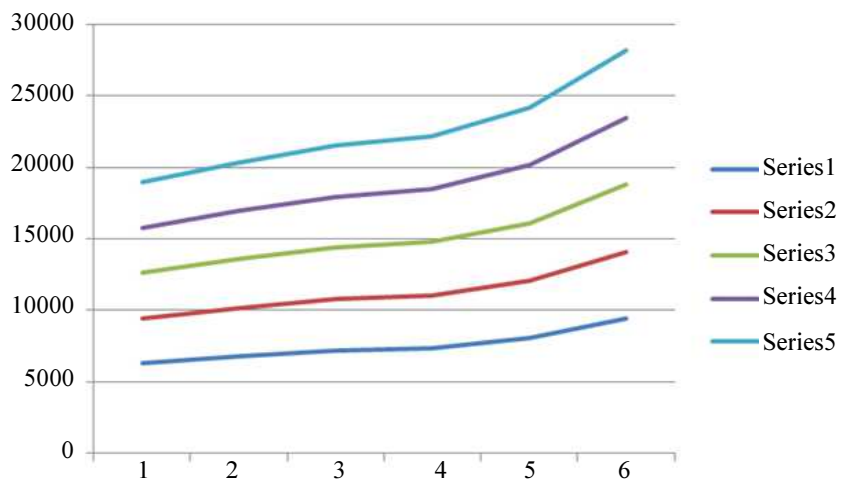**Fig. 2:** Plot of 75, 85 and 85, 85 columns of Table 3

Fig. 3: Annual rental marginal across age groups plot of rows of Table 3
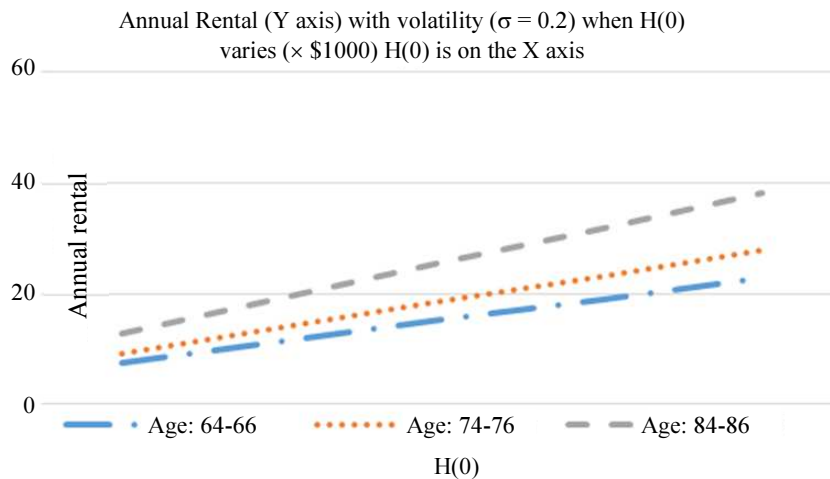
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Fig. 4: Annual rental summary of all couple pairs analysed

We may rearrange Table 3 in way that is increasing age combinations by swapping the 75, 85 and 75, 75 columns (in Table 3) and such a table may show a better picture in that we may compare the increases along rows and columns with increasing age more appropriately as shown in Table 4.

Figure 3 shows the annual rental calculations based on modelling increasing across and down the Table 4 in approximately a linear pattern initially with lower aged couples; and then at higher home prices and higher aged couples, the rental amount yearly becomes a non-linear and approaches a quadratic relationship.

In the Fig. 3 and 4 above, show a better summary of the earlier analysis is presented. There seems to be a particular trend line for the age group at which the couples retire. In this case, the x axis is the initial point real estate valuation of the home ($H(0)$) owned by a possible retiree. In most Australian cities the Bank valuations of houses range from 200K to 600K for lower to the middle value range or group in the housing market. In this analysis, the homes valued under 200K are not considered as these would not be suitable for the comparable rental housing needed for retirement of aged couples considered here; since it is assumed that the retirees own their own home in the analyses in this study. There is of course the condition of a single person condition as well but this may indeed become a different scenario given a single person may be able to stay in a unit of one bedroom for example and the valuation may indeed be less than 200K but this is an unlikely situation in the current housing market in Australia.

A Comparative Analysis of Renting in Australian Major Cities

There is much data in the literature regarding the Australian housing market and much of this data is available now on the web (https://www.ahuri.edu.au/).

Using real estate.com web site data (and others such as https://www.reiv.com.au/property-data/median-prices/median-prices-in-regional-victoria), we can calculate various rental housing statistics for major cities in Australia. For example, for the greater Melbourne the available homes/units for rent is about 2146 properties. The most expensive rental property is around $2500 per week ($130000 yearly); and the least expensive rental is $330 per week ($17160 yearly for a 1 bedroom unit). For greater Brisbane, the availability is around 7800; the most expensive rental is $2950 per week ($153400 yearly); and the least expensive rental is $110 per week ($5720 yearly for room – house share). In the case of greater Perth the availability for rent properties is around 12,785; with the most expensive rental is $3000 per week ($156000 yearly); and the least expensive rental is noted to be $70 per week ($3640 for room – house share)

In greater Hobart there are 265 properties available for rental with the most expensive being around $1000 per week ($52000 yearly); the least expensive rental is $130 per week ($6760 yearly for a 1 bed studio). In greater Darwin there are around 965 properties available for rent with the most expensive rental around $2000 per week ($104000 yearly); and the least expensive rental being around $179 per week ($9308 for a room in a house share condition). The lower rental availability (low supply) shows why the rents are rising in Hobart and also similarly but oppositely, with the large availability or supply is one of the reasons why Perth rental is lower.

The above data is only for 2016 and is the values are always changing, so the retirees may consider moving to different parts of Australia to retire for their benefit; if of course the market rental is much cheaper than the option of a RM contract. If a person owns their own home without mortgage when considering retirement and if the option that is favored by modelling is the rental rather than reverse mortgage then the person needs to consider

where to live for optimal financial condition or indeed optimal "choice" or "happiness" condition) and these two interests may help in the decision to choose an RM over market rentals in choice living locations. The information and statistics regarding the market rentals in this regard are available on various websites such as: https://www.reiv.com.au/property-data/rental-data/vacancy-rates.

Clearly, there is a range of values of rental prices in Australia and the rental values are usually dependent on locations such as whether the rental are units or houses and whether they are close to near or indeed in the city boundaries; or in the greater regions of cities; or whether in fact the rental are in regional areas of the states of Australia. The choice of retirement locations will play a major part in deciding whether a retiree may sell their home or move to other locations of choice, in which case reverse mortgage may not be as useful to them as to others who wish to remain in their homes for their retirements.

However, depending on conditions of a contract it may still be possible to move to other areas or locations of choice such as coastal regions and still engage in a reverse mortgage contract by allowing their off springs or indeed younger families to live in the homes during the period of the mortgage. This may be able to facilitate the present crises of housing for the younger generation. The problems of young buyers not being able to afford and thus not engage in the housing market is a major one for Australia. That is, it may be that taking a reverse mortgage may be not only beneficial to the retiree but also positively benefit the present housing conditions of supply etc., where in the main the young buyers are not able to buy into the 2016-17 market conditions.

Table 5 shows a summary of weekly rental for most major cities in Australia. The change is also given per year so some simplistic predictions can be made regarding the cities and trends in their rental prices.

The information in Table 5 is highly reliable since the Core Logic group argue that it is indeed "the largest provider of property information, analytics and property-related risk management services in Australia and New Zealand…." (see https://www.corelogic.com.au/). The range of rental in major cities is $359 to $593 per week in 2016; that translates to $18668 and $30836 yearly respectively. The average of all Australian cities combined is $481 per week, which translates to $25012 yearly.

The rental data gathered from the main cities suggest that Sydney is most expensive but there are equally areas that are similar to average rentals in other cities so in this study the rental data from all Sydney suburbs is analysed first and compared with the rentals generated from the our predicted reverse mortgage rental amounts using our mathematical model. This is because we need to examine the most populated market to determine whether it is reasonable to engage in RM; in the high market rental conditions. The RM will also be useful for all retirees in other less expensive rental markets because it could possibly be that they may sell their house and meet the lower market rental conditions but with our results and Australian housing rental data this seems unlikely (Fig. 5).

Using the median rental in Sydney the maximum yearly amount in dollars can be said to be around 36400 dollars or $700/week (Fig. 6). It is noted that the reverse mortgage modelled rental calculated in Tables 1-3 earlier shows that the prospective retirees may be much better off using their homes as a basis for rental consideration instead and thus initiate a RM process; taking a rental amount from the banks (as salary) who offer the reverse mortgage in accord to the valuations of the homes of retirees varying from 200k to 600k (Table 3).
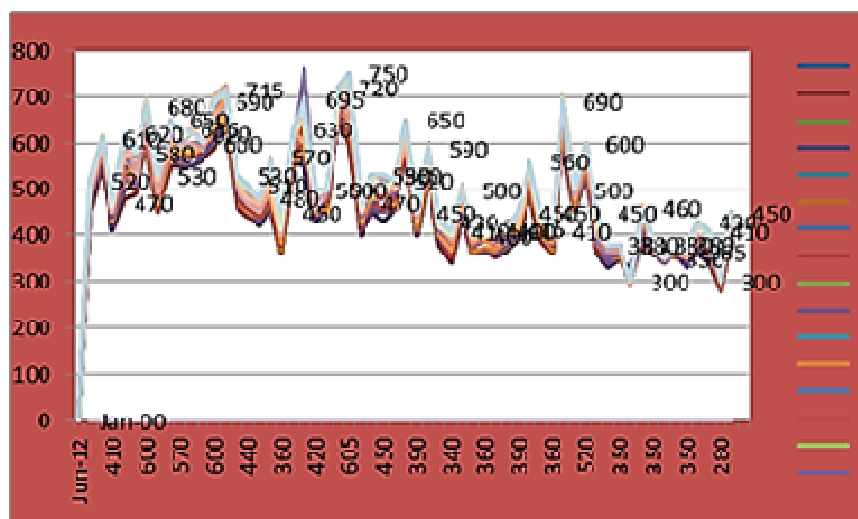

Fig. 5: Range of rental costs per week in Sydney (Houses and Units-Average rental values)

Bedroom Rental Scenario - Sydney

In Fig. 3 the x axis represents suburbs, where the values of 1 to 4 represent the inner city suburbs where in most cases the retirees will not prefer on average preference of a location for example. The outer suburbs are greater numbers from 4 and could represent a more realistic market rental option for a 2-Bedroom Unit. According to the amounts shown in Fig. 7, we can conclude that a 2 bedroom unit has a maximum rental of around 750 dollars per week (at the most expensive end), while the minimum cost is around 250 per week (at the least expensive end of the market). In the case of 700 a week, the yearly rental amount amounts to 39000 dollars; while the minimum is around 13000 dollars per

year. Now if we compare this to the rental amounts yearly using the Tables presented earlier.

In the first column of Table 4, we can conclude that fir a 65, 65 couple it may be better to undertake RM in the case of up to homes valued around 400K or less; but with home values greater than this it may be not as good for the market rental may be a cheaper option. For a 65, 75 couple, it may be an option to undertake market rental if their home is less than 400K or thereabouts (and less). For the 65, 85 couple, the RM would be better if their home was valued around 300K or less. For a 75, 75 (and a 75, 85 couple), the RM would be a better option if their home was valued around 300K less; and for an 85, 85 couple, the RM option is only appropriate if the home value is around 200K or less.
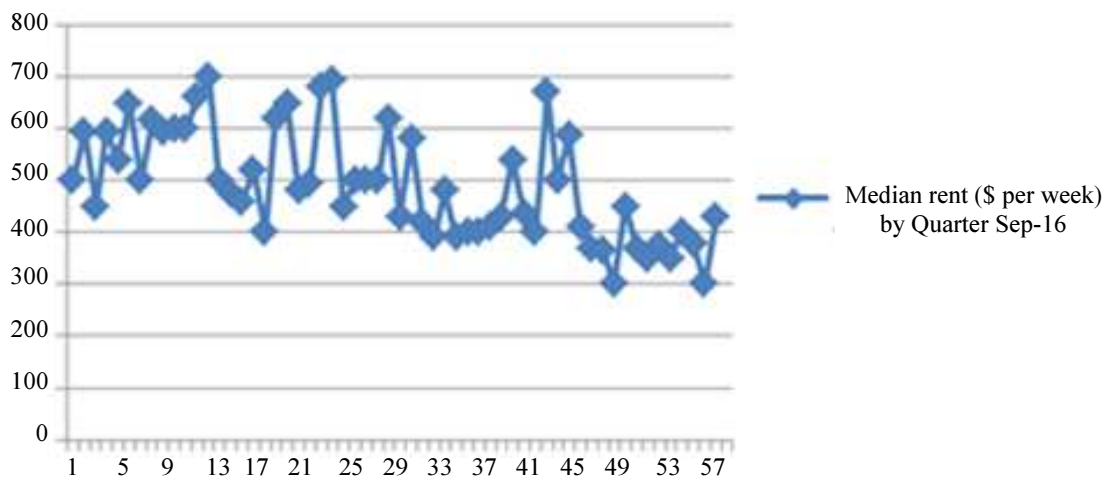
Fig. 6: Median Rental per week across all suburbs of Sydney (Houses and Units)
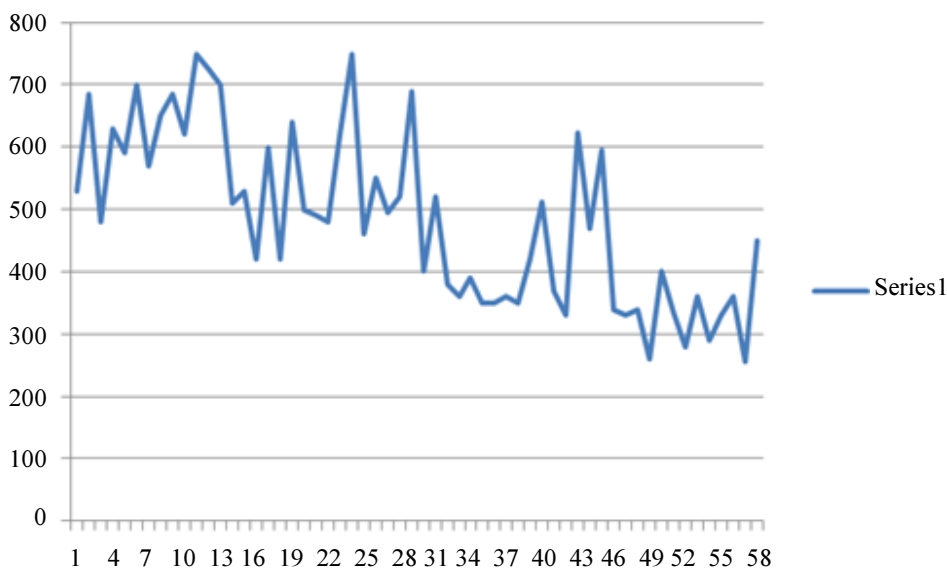
Fig. 7: Rental of 2-bedroom units in Sydney by suburbs (1-4 inner city; final average of NSW)

Table 5: House rental prices in major cities in Australia as at August 2016

City	Average weekly rent	% change 12 months
Sydney	$593	0.2
Melbourne	$458	+2.4
Brisbance	$427	-1.1
Perth	$419	-9.4
Adelaide	$364	-0.3
Canberra	$504	+2.6
Hobart	$359	+6.8
Darwin	$463	-14.1
Combined capitals	$481	-0.5

Discussion

Table 4 shows that for the 65, 65 age couples, it is most probably better to remain in the home and for retiring purpose proceed to invest in a reverse mortgage since the amount of rental yearly for their home is much lower than that available for a similar in the market for retirement purposes. The rental price for a 2-Bedroom Unit or a similar of real estate value house of around 200K-400K, is around 300 a week (as a minimum rental), which translates to $15600 yearly. This amount is higher yearly if the couples (choose to rent in a reasonable home. In most cases, the rental is around $450 a week and this translates to $23400 yearly for a modest home or unit possibly away from the central city. When compared to even a 600K value, the rental is still lower than the current rental market for a downsized 2-Bedroom unit or a 3 Bedroom home at least 30km from the city centre for example.

Melbourne House Rental Scenario

Using the above figures in Table 6 we may also suggest yearly averages of around $22646 in metro area and $15860 in regional areas for housing rental. Based on regional rentals and using the second major Australian city housing rental figures, we also conclude that a 65, 65 couple with home value of 500K or less could choose RM instead of seeking market rentals; while a 65, 75 couple with a home value of 400K or less may also choose to engage in RM; similarly, a 65, 85 couple could engage in RM if they have a house worth 400K or less. The 75, 85 aged couple should choose an RM if they own a home worth 300K or less; while the 75, 75 aged couples could choose RM if their home is worth 400K or less; and finally, the 85, 85 group could choose RM is their home is worth 300K or less.

In the case of choosing metro rental we can also make predictions of which age group couples should choose to remain in their property and engage in RM for their retirements. For example, in the case of Melbourne city, apart from 75, 85 and 85, 85 aged couples all the rest could choose to engage in RM if their homes were valued at 600K or less. In other words, it would be worth their while to be engaged in a RM contract and remain in their homes during retirement because in essence they will be paying less rental amounts when compared to market conditions.

Table 6: Median house rental (per week) in Melbourne

Melbourne	Median Rent (Houses)
METRO	$435.5
REGIONAL	$305

Summary and Conclusion

The aim of this paper was to compare the rental market conditions in retirement with an RM contract that involves a retiree's home and its valuation. It is noted that the current rental market conditions is to be examined thoroughly before any engagement in RM is to be instituted. The rental value per week for 1, 2-Bedroom Units and 3 or more Bedroom homes are to study in some depth. The availability of rental housing should also be studied to determine whether it is indeed possible to obtain secure longer term rental housing for the retired persons. In this study only a certain group has been investigated and further work is necessary if RM is to be a major contract that retired persons may engage in thus achieving a better quality of life after retirement given the many years of service couples have given to their country.

The human nature of unknown, or not knowing what may be involved in terms of details within a reverse mortgage contract including many other complexities that can be involved in releasing one's ownership of to the reverse mortgage contract has meant that many countries are now lagging behind the more advanced countries such as the US, Singapore and others in delving into the reverse mortgage market focusing on retired persons. This is clearly noted in the major lack of information that is able in Australia for example. There is a clear lack of discussion of RM's amongst the markets and institutions operators such as credit companies and banks for example. There is indeed hardly if any advertising of RM in the major forms of the media. It is true that RM is less strongly pushed even in the usual transactions in banks when often products such as insurance and the like are loudly announced often as you enter many banks; mainly pushed by their tellers on site. The lack of engagement in Australia is a two way story in that the less than usual interest of retiree's on RM is met by the banks very low level of advertising of the same. The lack of information has created a lack of trust in RM; the thought of letting someone else own

apart of one's home towards the later part of one's life in some manner is not well accepted or understood given the retiree's home is mostly a mortgage free asset.

The modelling and analyses conducted in this study shows that the return in terms of rental is well in competition with the existing rental market in Australia and in particular when using the largest city data from Sydney and other rental markets.

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Author Contribution

Gurudeo Anand Tularam: 60%
Ali Abdullah Al Ibrahim: 40%

Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and there are no ethical issues involved.

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